5.1 - Linear Models: Initial-Value Problems

5.1a Mass-Spring Models Setup

5.1b

Undamped motion

Example: A mass weighing 32 pounds stretches a spring 2 feet. Determine the amplitude and period of motion if the mass is initially released from a point 1 foot above the equilibrium position with an upward velocity of 2 ft/s. How many complete cycles will the mass have completed at the end of 4π seconds?

5.1c

Multiple springs in parallel: $k_{\text{eff}} = k_1 + k_2$

Multiple springs in series: $k_{\text{eff}} = \dfrac{k_1 k_2}{k_1 + k_2}$

Damped motion

$$m\frac{d^2x}{dt^2} = -kx - \beta\frac{dx}{dt}$$

Driven motion

$$m\frac{d^2x}{dt^2} + kx + \beta\frac{dx}{dt} = f(t)$$

5.1d

LRC-series circuit

$L\frac{di}{dt} + Ri + \frac{1}{C}q = E(t)$, which is

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{1}{C}q = E(t)$$

Example: Find the charge on the capacitor in an LRC-series circuit when $L = \frac{1}{4}$ h, $R = 20\ \Omega$, $C = \frac{1}{300}$ f, $E(t) = 0$ V, $q(0) = 4$ C, and $i(0) = 0$ A. Is the charge on the capacitor ever equal to zero?