4.3 - Homogeneous Linear Equations with Constant Coefficients

4.3a

Consider the differential equation $y'' - 5y' + 6y = 0$.

Definition: The **auxiliary equation** of the second-order linear differential equation $ay'' + by' + cy = 0$ is $am^2 + bm + c = 0$.

4.3b

Example: Find the general solution of the given second-order differential equation.

$y'' - 10y' + 25y = 0$

Example: $2y'' - 3y' + 4y = 0$

Example: Find the general solution of the given higher-order differential equation.

$$\frac{d^3x}{dt^3} - \frac{d^2x}{dt^2} - 4x = 0$$

Special 2nd-order differential equations

$y'' + k^2y = 0$ and $y'' - k^2y = 0, k \in \mathbb{R}$

4.3c

Example: Find a homogeneous linear differential equation with constant coefficients whose general solution is given.

$y = c_1 e^{-4x} + c_2 e^{-3x}$

Example: $y = c_1 e^{10x} + c_2 x e^{10x}$

Example: $y = c_1 + c_2 e^{2x} \cos 5x + c_3 e^{2x} \sin 5x$