3.2 - Nonlinear Models

3.2a

Logistic growth

$$\frac{dP}{dt} = P(a - bP)$$

3.2b

Modified logistic growth

$$\frac{dP}{dt} = P(a - bP) \pm h$$

Example: (a) If a constant number h of fish are harvested from a fishery per unit time, then a model for the population $P(t)$ of the fishery at time t is given by $\frac{dP}{dt} = P(a - bP) - h$, $P(0) = P_0$, where a, b, h, and P_0 are positive constants. Suppose $a = 5$, $b = 1$, and $h = 4$. Since the DE is autonomous, use the phase portrait concept of Section 2.1 to sketch representative solution curves corresponding to the cases $P_0 > 4$, $1 < P_0 < 4$, and $0 < P_0 < 1$. Determine the long-term behavior of the population in each case.

(b) Solve the IVP in part (a). Verify the results of your phase portrait in part (a) by using a graphing utility to plot the graph of $P(t)$ with an initial condition taken

from each of the three intervals given.

(c) Use the information in parts (a) and (b) to determine whether the fishery population becomes extinct in finite time. If so, find that time.

3.2c

Second-order chemical reaction

$\frac{dX}{dt} = k(\alpha - X)(\beta - X)$,

where $\alpha = \frac{a(M+N)}{M}$ and $\beta = \frac{b(M+N)}{N}$,

$a =$ # grams of chemical A,

$b =$ # grams of chemical B, and

$X(t)$ is grams of C, formed from M parts of A and N parts of B

Example: Two chemicals A and B are combined to form a chemical C. The rate, or velocity, of the reaction is proportional to the product of the instantaneous amounts of A and B not converted to chemical C. Initially, there are 80 grams of A and 50 grams of B, and for each gram of B, 2 grams of A is used. It is observed that 10 grams of C is formed in 10 minutes. How much is formed in 40 minutes? What is the limiting amount of C after a long time? How much of chemicals A and B remains after a long time?