2.5 - Solutions by Substitutions

2.5a

We will use substitutions to solve DEs (and IVPs) of the following types.

$\left(y^2 + xy\right)\, dx + x^2\, dy = 0$

An equation of the form $M(x, y)\, dx + N(x, y)\, dy = 0$ is a homogeneous equation, if the coefficient functions possess the property $f(tx, ty) = t^{\alpha} f(x, y)$ (with the same α for both functions).

Substitution:

$\frac{dy}{dx} - y = e^x y^2$

The DE $\frac{dy}{dx} + P(x)y = f(x)y^n$ is called Bernoulli's equation.

Substitution:

The equation $\frac{dy}{dx} = f(Ax + By + C)$ can be converted to a separable equation using the substitution $u = Ax + By + C$.

$$\frac{dy}{dx} = \frac{3x + 2y}{3x + 2y + 2}$$

2.5b

Example: Solve the given differential equation by using an appropriate substitution.

$\left(y^2 + xy\right)\, dx + x^2\, dy = 0$

2.5c

Example: $\frac{dy}{dx} - y = e^x y^2$

Example: $3\left(1+t^2\right)\dfrac{dy}{dt}=2ty\left(y^3-1\right)$

2.5d

Example: Solve the given initial-value problem.

$$\frac{dy}{dx} = \frac{3x + 2y}{3x + 2y + 2}, \qquad y(-1) = -1$$