2.3 - Linear Equations

2.3a

Definition: A first-order differential equation of the form $a_1(x)\frac{dy}{dx} + a_0(x)y = g(x)$ is a **linear equation** in y (the dependent variable). This is put into **standard form** by dividing both sides by $a_1(x)$ to obtain

$$\frac{dy}{dx} + P(x)y = f(x)$$

Example: Consider the DE $3\frac{dy}{dx} + 12y = 4$. In standard form, this is $\frac{dy}{dx} + 4y = \frac{4}{3}$.

Where did e^{4x} come from?

2.3b

Example: Find the general solution of the given differential equation. Give the largest interval I over which the general solution is defined. Determine whether there are any transient terms in the general solution.

$y' + 2xy = x^3$

Example: $(1+x)\dfrac{dy}{dx} - xy = x + x^2$

2.3c

Example: $xy' + (1 + x)y = e^{-x} \sin 2x$

Example: $\frac{dP}{dt} + 2tP = P + 4t - 2$

Example: $(x^2 - 1)\frac{dy}{dx} + 2y = (x + 1)^2$

2.3d

Example: Solve the given initial-value problem. Use a graphing utility to graph the continuous function $y(x)$.

$(1 + x^2)\dfrac{dy}{dx} + 2xy = f(x), \qquad y(0) = 0$, where

$$f(x) = \begin{cases} x, & 0 \le x < 1 \\ -x, & x \ge 1 \end{cases}$$