8.2 - Homogeneous Linear Systems

8.2a - Matrix Preliminaries

8.2b - Framework for solutions using matrices

Consider the system $\begin{aligned} \frac{dx}{dt} &= 2x + 2y \\ \frac{dy}{dt} &= x + 3y \end{aligned}$ which, in $\mathbf{X}' = \mathbf{AX}$ form is

$$\mathbf{X}' = \begin{pmatrix} 2 & 2 \\ 1 & 3 \end{pmatrix} \mathbf{X}.$$

8.2c

Example: Find the general solution of the given system.

$$\frac{dx}{dt} = 2x + 2y$$
$$\frac{dy}{dt} = x + 3y$$

8.2d

Example: Find the general solution of the given system.

$$\frac{dx}{dt} = 2x - 7y$$
$$\frac{dy}{dt} = 5x + 10y + 4z$$
$$\frac{dz}{dt} = 5y + 2z$$

8.2e - Eigenvalue possibilities, derivation of sine/cosine solutions, example

Theorem: Let $\lambda = \alpha + i\beta$ be an eigenvalue of $\mathbf{A}$ and let $\mathbf{B}_1 = \text{Re}(\mathbf{K})$, $\mathbf{B}_2 = \text{Im}(\mathbf{K})$. Then

$$\mathbf{X}_1 = e^{\alpha t}(\mathbf{B}_1 \cos \beta t - \mathbf{B}_2 \sin \beta t)$$
$$\mathbf{X}_2 = e^{\alpha t}(\mathbf{B}_2 \cos \beta t + \mathbf{B}_1 \sin \beta t)$$

Example: Find the general solution of the given system.

$$\frac{dx}{dt} = 4x + 5y$$
$$\frac{dy}{dt} = -2x + 6y$$

8.2f

Repeated eigenvalues:

Example: Find the general solution of the given system.

$$\mathbf{X}' = \begin{pmatrix} 12 & -9 \\ 4 & 0 \end{pmatrix} \mathbf{X}$$

Example: Find the general solution of the given system.

8.2g

$$\frac{dx}{dt} = 3x + 2y + 4z$$
$$\frac{dy}{dt} = 2x + 2z$$
$$\frac{dz}{dt} = 4x + 2y + 3z$$