7.5 - The Dirac Delta Function

7.5a

Definition: the unit impulse function is defined as

$$\delta_a(t-t_0) = \begin{cases} 0, & 0 \le t < t_0 - a \\ \dfrac{1}{2a}, & t_0 - a \le t < t_0 + a \\ 0, & t \ge t_0 + a \end{cases}$$

Definition: The **Dirac delta "function"** is $\delta(t - t_0) = \lim_{a \to 0} \delta_a(t - t_0)$.

Theorem: Transform of the Dirac Delta Function

For $t_0 > 0$, $\quad \mathscr{L}\{\delta(t - t_0)\} = e^{-st_0}$.

Note that when $t_0 = 0$, we have $\mathscr{L}\{\delta(t)\} = 1$.

7.5b

Example: Use the Laplace transform to solve the given initial-value problem.

$y'' + y = \delta(t - 2\pi) + \delta(t - 4\pi), \qquad y(0) = 1, \quad y'(0) = 0$

$y'' - 7y' + 6y = e^t + \delta(t - 2) + \delta(t - 4), \qquad y(0) = 0, \quad y'(0) = 0$