

7.2 - Inverse Transforms and Transforms of Derivatives

7.2a

Theorem 7.2.1: Some Inverse Transforms

$$(a) 1 = \mathcal{L}^{-1} \left\{ \frac{1}{s} \right\}$$

$$(b) t^n = \mathcal{L}^{-1} \left\{ \frac{n!}{s^{n+1}} \right\}, n = 1, 2, 3, \dots$$

$$(c) e^{at} = \mathcal{L}^{-1} \left\{ \frac{1}{s-a} \right\}$$

$$(d) \sin kt = \mathcal{L}^{-1} \left\{ \frac{k}{s^2 + k^2} \right\}$$

$$(e) \cos kt = \mathcal{L}^{-1} \left\{ \frac{s}{s^2 + k^2} \right\}$$

$$(f) \sinh kt = \mathcal{L}^{-1} \left\{ \frac{k}{s^2 - k^2} \right\}$$

$$(g) \cosh kt = \mathcal{L}^{-1} \left\{ \frac{s}{s^2 - k^2} \right\}$$

Example: Find the inverse Laplace transform.

$$\mathcal{L}^{-1} \left\{ \frac{1}{s^4} \right\}$$

$$\mathcal{L}^{-1} \left\{ \frac{10s}{s^2 + 16} \right\}$$

$$\mathcal{L}^{-1} \left\{ \frac{s+1}{s^2+2} \right\}$$

$$\mathcal{L}^{-1} \left\{ \frac{s+1}{s^2-4s} \right\}$$

7.2b

Example: Use the Laplace transform to solve the Initial-Value Problem.

$$2 \frac{dy}{dt} + y = 0, \quad y(0) = -3$$

Transforms of derivatives



7.2c

Theorem: Transform of a Derivative

If $f, f', \dots, f^{(n-1)}$ are continuous on $[0, \infty)$ and are of exponential order and if $f^{(n)}(t)$ is piecewise continuous on $[0, \infty)$, then

$$\mathcal{L}\{f^{(n)}(t)\} = s^n F(s) - s^{n-1}f(0) - s^{n-2}f'(0) - \dots - f^{(n-1)}(0),$$

where $F(s) = \mathcal{L}\{f(t)\}$.

Example: Use the Laplace transform to solve the Initial-Value Problem.

$$y'' - 4y' = 6e^{3t} - 3e^{-t}, \quad y(0) = 1, \quad y'(0) = -1$$

