6.2 - Solutions About Ordinary Points

6.2a

Consider the differential equation $a_2(x)y'' + a_1(x)y' + a_0(x)y = 0$, which in standard form is $y'' + P(x)y' + Q(x)y = 0$.

Definition: A point $x = x_0$ is an **ordinary point** if P and Q are analytic at x_0 (meaning power series representations centered at $x = x_0$) exist. A point that is not ordinary is **singular**.

Theorem: If $x = x_0$ is an ordinary point of
$a_2(x)y''+a_1(x)y'+a_0(x)y = 0$, then we can always find two linearly independent solutions of the form $y = \sum_{n=0}^{\infty} c_n(x - x_0)^n$.

Example: Find two power series solutions of the given differential equation about the ordinary point $x = 0$.

$y'' + 2xy' + 2y = 0$

6.2b

Example: Find two power series solutions of the given differential equation about the ordinary point $x = 0$.

$(x + 2)y'' + xy' - y = 0$

6.2c

Example: Find two power series solutions of the given differential equation about the ordinary point $x = 0$.

$y'' - (x + 1)y' - y = 0$

6.2d

Use an appropriate Maclaurin series to find two power series solutions of the given differential equation about the ordinary point $x = 0$.

$y'' + e^x y' - y = 0$