4.9 - Solving Systems of Linear DEs by Elimination

4.9a - Working with the differential operator

4.9b and 4.9c

The DE $y' + 2y + 3x = -1$ can be written in terms of the differential operator D, where D represents a derivative. In this form, the DE becomes $Dy + 2y + 3x = -1$. This can also be written $(D+2)y + 3x = -1$. Our work in this lesson makes use of this notation. For this lesson, x and y are functions of t, so $Dx = \frac{dx}{dt}$, $Dy = \frac{dy}{dt}$, and $D^2x = \frac{d^2x}{dt^2}$, etc.

Solve the given system of differential equations by elimination.

Example:

$$\begin{aligned} (D+1)x &+ (D-1)y &&= 2 \\ 3x &+ (D+2)y &&= -1 \end{aligned}$$

4.9d

Example:

$$\frac{dx}{dt} + \frac{dy}{dt} = e^t$$

$$-\frac{d^2x}{dt^2} + \frac{dx}{dt} + x + y = 0$$

Example:

$$\begin{aligned} Dx \quad &+z &&= e^t \\ (D-1)x \quad &+Dy \quad &+Dz \quad &= 0 \\ x \quad &+2y \quad &+Dz \quad &= e^t \end{aligned}$$

4.9e

Example:

$$\frac{dx}{dt} = -x + z$$

$$\frac{dy}{dt} = -y + z$$

$$\frac{dz}{dt} = -x + y$$