4.6 - Variation of Parameters

4.6a

Consider the homogeneous second-order differential equation $a_2(x)y'' + a_1(x)y' + a_0(x)y = 0$. The general solution to this DE is $y = c_1y_1 + c_2y_2$. When the input function of an associated nonhomogeneous DE is not restricted to the type we saw in 4.4, we consider particular solutions with coefficients that are variable functions. That is, we will find $y_p(x) = u_1(x)y_1(x) + u_2(x)y_2(x)$.

4.6b

$$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}, \qquad W_1 = \begin{vmatrix} 0 & y_2 \\ f(x) & y_2' \end{vmatrix}, \qquad W_2 = \begin{vmatrix} y_1 & 0 \\ y_1' & f(x) \end{vmatrix}$$

$$u_1' = \frac{W_1}{W} \implies u_1 = \int \frac{W_1}{W} dx$$
$$u_2' = \frac{W_2}{W} \implies u_2 = \int \frac{W_2}{W} dx$$

$y_p = u_1 y_1 + u_2 y_2$ and $y = y_c + y_p$

Example: Solve each differential equation by variation of parameters.
$y'' + y = \sec\theta \ \tan\theta$

4.6c

Example: Solve the initial-value problem by variation of parameters.

$2y'' + y' - y = x + 1 \qquad y(0) = 1,\ y'(0) = 0$

Example: Solve the differential equation by variation of parameters.

$$y'' - 4y = \frac{e^{2x}}{x}$$

4.6d

Example: Solve the given third-order differential equation by variation of parameters.

$y''' + 4y' = \sec 2x$