3.1 - Linear Models

3.1a

Various situations are modeled by differential equations.

- Growth and decay: $\dfrac{dx}{dt} = kx, \qquad x(t_0) = x_0$

- Newton's law of heating and cooling: $\dfrac{dT}{dt} = k(T - T_m), \qquad T(t_0) = T_0$

- Mixtures: $\dfrac{dA}{dt} = \text{rate in} - \text{rate out}, \qquad A(0) = A_0$

- LR-series electrical circuit: $L\dfrac{di}{dt} + Ri = E(t)$

- RC-series electrical circuit: $R\dfrac{dq}{dt} + \dfrac{1}{C}q = E(t)$

Example: A population of bacteria in a culture grows at a rate proportional to the number of bacteria present at time t. After 3 hours it is observed that 400 bacteria are present. After 10 hours 2000 bacteria are present. What was the initial number of bacteria?

3.1b

Example: A thermometer is taken from an inside room to the outside, where the air temperature is 5° F. After 1 minute the thermometer reads 55° F, and after 5 minutes it reads 30° F. What is the initial temperature of the inside room?

3.1c

Example: A large tank is filled to capacity with 500 gallons of pure water. Brine containing 2 pounds of salt per gallon is pumped into the tank at a rate of 5 gal/min. The well-mixed solution is pumped out at the same rate. Find the number $A(t)$ of pounds of salt in the tank at time t.

Example: Solve the previous problem under the assumption that the solution is pumped out a a faster rate of 10 gal/min. When is the tank empty?

Example: A 200-volt electromotive force is applied to an RC-series circuit in which the resistance is 1000 ohms and the capacitance is 5×10^{-6} farad. Find the charge $q(t)$ on the capacitor if $i(0) = 0.4$. Determine the charge and current at $t = 0.005$ s. Determine the charge as $t \to \infty$.