

2.5 - Solutions by Substitutions

2.5a

We will use substitutions to solve DEs (and IVPs) of the following types.

$$(y^2 + xy) dx + x^2 dy = 0$$

An equation of the form $M(x, y) dx + N(x, y) dy = 0$ is a homogeneous equation, if the coefficient functions possess the property $f(tx, ty) = t^\alpha f(x, y)$ (with the same α for both functions).

Substitution:

$$\frac{dy}{dx} - y = e^x y^2$$

The DE $\frac{dy}{dx} + P(x)y = f(x)y^n$ is called Bernoulli's equation.

Substitution:

The equation $\frac{dy}{dx} = f(Ax + By + C)$ can be converted to a separable equation using the substitution $u = Ax + By + C$.

2.5b

Example: Solve the given differential by using an appropriate substitution.

$$(y^2 + xy) \, dx + x^2 \, dy = 0$$

[illegible]

2.5c

Example: $\frac{dy}{dx} - y = e^x y^2$

[illegible]

2.5d

Example: Solve the given initial-value problem.

$$\frac{dy}{dx} = \frac{3x + 2y}{3x + 2y + 2}, \quad y(-1) = -1$$