1.2 – Gaussian Elimination

#7 Solve the system by Gaussian elimination.

$$\begin{aligned} x - y + 2z - w &= -1 \\ 2x + y - 2z - 2w &= -2 \\ -x + 2y - 4z + w &= 1 \\ 3x \qquad\quad - 3w &= -3 \end{aligned}$$

Definitions: **Row echelon form** (#1-3 below) and **reduced row echelon form** (#1-4 below) of a matrix – to be of this form, a matrix must have the following properties:

1. If a row does not consist entirely of zeros, then the first nonzero number in the row is a 1. This is a **leading 1.**
2. If there are any rows that consist entirely of zeros, then they are grouped together at the bottom of the matrix.
3. In any two successive rows that do not consist entirely of zeros, the leading 1 in the lower row occurs farther to the right than the leading 1 in the higher row.
4. Each column that contains a leading 1 has zeros everywhere else in that column.

Definitions: **Leading variables** correspond to the leading 1's in the augmented matrix for a linear system. **Free variables** are the remaining variables in the linear system.

Definition: In a **homogeneous linear system**, the constant terms are all zero. That is, it has the form

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n &= 0 \\ a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n &= 0 \\ \vdots \qquad\qquad \vdots \qquad\qquad & \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}x_n &= 0 \end{aligned}$$

Theorem 1.2.1 **Free Variable Theorem for Homogeneous Systems**
If a homogeneous linear system has n unknowns, and if the reduced row echelon form of its augmented matrix has r nonzero rows, then the system has $n - r$ free variables.

Theorem 1.2.2 A homogeneous linear system with more unknowns than equations has infinitely many solutions.

#28 What condition, if any, must a, b, and c satisfy for the linear system to be consistent?

$$\begin{aligned} x + 3y + z &= a \\ -x - 2y + z &= b \\ 3x + 7y - z &= c \end{aligned}$$

Solve the given linear system by any method.

#16

$$\begin{aligned} 2x - y - 3z &= 0 \\ -x + 2y - 3z &= 0 \\ x + y + 4z &= 0 \end{aligned}$$

#19

$$\begin{aligned} 2x + 2y + 4z &= 0 \\ w - y - 3z &= 0 \\ 2w + 3x + y + z &= 0 \\ -2w + x + 3y - 2z &= 0 \end{aligned}$$

#23 The augmented matrix for a linear system is given in which the asterisk represents an unspecified real number. Determine whether the system is consistent, and if so whether the solution is unique. Answer "inconclusive" if there is not enough information to make a decision.

a. $\begin{bmatrix} 1 & * & * & * \\ 0 & 1 & * & * \\ 0 & 0 & 1 & * \end{bmatrix}$

b. $\begin{bmatrix} 1 & * & * & * \\ 0 & 1 & * & * \\ 0 & 0 & 0 & 0 \end{bmatrix}$

c. $\begin{bmatrix} 1 & * & * & * \\ 0 & 1 & * & * \\ 0 & 0 & 0 & 1 \end{bmatrix}$

d. $\begin{bmatrix} 1 & * & * & * \\ 0 & 0 & * & 0 \\ 0 & 0 & 1 & * \end{bmatrix}$