

Change of Variables Summary

- The double integral in polar coordinates
 - $\iint_R f(x, y) dA = \int_{\alpha}^{\beta} \int_a^b f(r \cos \theta, r \sin \theta) r dr d\theta$
 - AND $\iint_D f(x, y) dA = \int_{\alpha}^{\beta} \int_{h_1(\theta)}^{h_2(\theta)} f(r \cos \theta, r \sin \theta) r dr d\theta$
 - Note that $dV = r dr d\theta$
- The triple integral in cylindrical coordinates
 - $\iiint_E f(x, y, z) dV = \int_{\alpha}^{\beta} \int_{h_1(\theta)}^{h_2(\theta)} \int_{u_1(r \cos \theta, r \sin \theta)}^{u_2(r \cos \theta, r \sin \theta)} f(r \cos \theta, r \sin \theta) r dz dr d\theta$
 - Note that $dA = r dz dr d\theta$
 - Typically used when you see $x^2 + y^2$, when D is circular, or when the integrand is easier in cylindrical coordinates
- The triple integral in spherical coordinates
 - $\iiint_E f(x, y, z) dV$

$$= \int_c^d \int_{\alpha}^{\beta} \int_{g_1(\theta, \phi)}^{g_2(\theta, \phi)} f(\rho \cos \theta \sin \phi, \rho \sin \theta \sin \phi, \rho \cos \phi) \rho^2 \sin \phi d\rho d\theta d\phi$$
 - Note that $dV = \rho^2 \sin \phi d\rho d\theta d\phi$
 - Typically used when you see $x^2 + y^2 + z^2$ or when E involves cones/spheres
- Change of variables for double integrals (under $T: S \rightarrow R$)
 - $\iint_R f(x, y) dA = \iint_S f(g(u, v), h(u, v)) \left| \frac{\partial(x, y)}{\partial(u, v)} \right| du dv$, where
 - $$\frac{\partial(x, y)}{\partial(u, v)} = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial y}{\partial u} \\ \frac{\partial x}{\partial v} & \frac{\partial y}{\partial v} \end{vmatrix}$$
 - $u = f(x, y)$ and $v = g(x, y)$ are useful in finding limits of integration
 - $x = x(u, v)$ and $y = y(u, v)$ are useful for finding $\frac{\partial(x, y)}{\partial(u, v)}$
 - Substitute using whichever is most convenient