

8.2 - Area of a Surface of Revolution

8.2a

Area of a surface of revolution

[illegible]

About the x -axis: $S = \int 2\pi y \, ds$

About the y -axis: $S = \int 2\pi x \, ds$

$ds = \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx = \sqrt{1 + \left(\frac{dx}{dy}\right)^2} dy$, whichever is most convenient

Limits of integration: $a \leq x \leq b$ for $y = f(x)$; for $c \leq y \leq d$ for $x = g(y)$

8.2b

Exercise:

Find the exact area of the surface obtained by rotating the curve about the x -axis.

$$y = \sqrt{1 + e^x}, 0 \leq x \leq 1$$

$$y = \frac{x^3}{6} + \frac{1}{2x}, \frac{1}{2} \leq x \leq 1$$

8.2c

Exercise:

The given curve is rotated about the y -axis. Find the area of the resulting surface.

$$x^{2/3} + y^{2/3} = 1, 0 \leq y \leq 1$$
