

8.1 - Arc Length

8.1a

This image shows a blank sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

Arc length formula for $y = f(x)$, $a \leq x \leq b$

$$L = \int_a^b \sqrt{1 + [f'(x)]^2} \, dx = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2} \, dx$$

8.1b

Exercises:

Set up an integral that represents the length of the curve. Then use your calculator to find the length correct to four decimal places.

$$y = xe^{-x}, 0 \leq x \leq 2$$

Find the exact length of the curve.

$$36y^2 = (x^2 - 4)^3, 2 \leq x \leq 3, y \geq 0$$

8.1c

Arc length function

$$s(x) = \int_a^x \sqrt{1 + [f'(t)]^2} dt$$

Two forms for ds

$$ds = \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx = \sqrt{1 + \left(\frac{dx}{dy}\right)^2} dy$$

Arc length formula for $x = g(y)$, $c \leq y \leq d$

$$L = \int_c^d \sqrt{1 + [g'(y)]^2} dy = \int_c^d \sqrt{1 + \left(\frac{dx}{dy}\right)^2} dy$$

Exercises:

Find the exact length of the curve.

$$x = \frac{y^4}{8} + \frac{1}{4y^2}, 1 \leq y \leq 2$$

8.1d

$$y = \ln(\cos x), 0 \leq x \leq \frac{\pi}{3}$$

$$y = 1 - e^{-x}, 0 \leq x \leq 2$$

