

7.8 - Improper Integrals

7.8a

Types of improper integrals

Infinite interval: $\int_{-\infty}^a f(x) dx = \lim_{t \rightarrow -\infty} \int_t^a f(x) dx$ or $\int_a^{\infty} f(x) dx = \lim_{t \rightarrow \infty} \int_a^t f(x) dx$

Infinite discontinuity: $\int_a^b f(x) dx$, where there is $c \in [a, b]$ such that $\lim_{x \rightarrow c} f(x) = \pm\infty$. In this case we have

$$\int_a^b f(x) dx = \lim_{t \rightarrow c^-} \int_a^t f(x) dx + \lim_{t \rightarrow c^+} \int_t^b f(x) dx$$

Exercises:

Determine whether each integral is convergent or divergent. Evaluate those that are convergent.

$$\int_1^{\infty} \frac{e^{-1/x}}{x^2} dx$$

$$\int_0^{\infty} \sin \theta e^{\cos \theta} d\theta$$

7.8b

Exercises:

Determine whether each integral is convergent or divergent. Evaluate those that are convergent.

$$\int_2^{\infty} \frac{dv}{v^2 + 2v - 3}$$

$$\int_0^1 \frac{dx}{\sqrt{1-x^2}}$$

[illegible]

For what values of p is the integral $\int_1^{\infty} \frac{1}{x^p} dx$ convergent?

7.8d

Theorem (statement):

Comparison Theorem: If $f(x) \geq g(x) \geq 0$ (both continuous) for $x \geq a$, then

a) If $\int_a^\infty f(x) \, dx$ is convergent, then so is $\int_a^\infty g(x) \, dx$

b) If $\int_a^\infty g(x) \, dx$ is divergent, then so is $\int_a^\infty f(x) \, dx$

Exercise:

Use the Comparison Theorem to determine whether the integral is convergent or divergent.

$$\int_1^\infty \frac{1 + \sin^2 x}{\sqrt{x}} \, dx$$
