7.7 - Approximate Integration

7.7a

The Midpoint Rule:

$\int_a^b f(x)\,dx \approx \Delta x[f(\overline{x}_1) + f(\overline{x}_2) + \cdots f(\overline{x}_n)]$, where $\Delta x = \dfrac{b-a}{n}$ and $\overline{x}_i = \dfrac{x_{i-1} + x_i}{2}$, the midpoint of $[x_{i-1},\ x_i]$

Error is $|E_M| \leq \dfrac{K(b-a)^3}{24n^2}$, where $\left|f''(x)\right| \leq K$ for $x \in [a,\ b]$

7.7b

The Trapezoid Rule:

$$\int_a^b f(x)\,dx \approx \frac{\Delta x}{2}\left[f(x_0) + 2f(x_1) + 2f(x_2) + \cdots 2f(x_{n-1}) + f(x_n)\right]$$, where

$\Delta x = \dfrac{b-a}{n}$ and $x_i = a + i\Delta x$

Error is $|E_T| \leq \dfrac{K(b-a)^3}{12n^2}$, where $\left|f''(x)\right| \leq K$ for $x \in [a,\ b]$

Exercises:

7.7c

Use (a) the Trapezoidal Rule and (b) the Midpoint Rule to approximate the given integral with the specified value of n. (Round your answers to six decimal places.)

$$\int_0^4 \sqrt{y} \cos y \, dy, \; n = 8$$

7.7d

a) Estimate the errors for T_{10} and M_{10} for $\int_1^2 e^{1/x}\,dx$.

b) How large do we have to choose n so that the approximations T_n and M_n to the integral in part (a) are accurate to within 0.0001?

7.7e

Simpson's Rule:

$$\int_a^b f(x)\,dx \approx \frac{\Delta x}{3}[f(x_0)+4f(x_1)+2f(x_2)+4f(x_3)+\cdots 2f(x_{n-2})+4f(x_{n-1})+$$

$f(x_n)]$, where n is even, $\Delta x = \dfrac{b-a}{n}$, and $x_i = a + i\Delta x$

Error is $|E_S| \le \dfrac{K(b-a)^5}{180n^4}$, where $\left|f^{(4)}(x)\right| \le K$ for $x \in [a,\ b]$

1. The following instructions pertain to the integral $I = \displaystyle\int_0^{2\pi} e^{\cos x}\,dx$. For this exercise, please use the following tools: WolframAlpha (https://www.wolframalpha.com

a graphing utility such as Desmos (https://www.desmos.com/calculator) or Geogebra (https://www.geogebra.org/graphing), and Excel (or another spreadsheet program).

a) Use a graph to get a good upper bound for $|f''(x)|$.

b) Use M_{10} to approximate I.

c) Use part (a) to estimate the error in part (b).

d) Use WolframAlpha to approximate I.

e) How does the actual error compare with the error estimate in part (c)?

f) Use a graph to get a good upper bound for $\left|f^{(4)}(x)\right|$.

g) Use S_{10} to approximate I.

h) Use part (f) to estimate the error in part (g).

i) How does the actual error compare with the error estimate in part (h)?

j) How large should n be to guarantee that the size of the error in using S_{10} is less than 0.0001?