7.4 - Integration of Rational Functions by Partial Fractions

7.4a

Start with a proper rational function. If the function is an improper rational function, use long division before starting the partial fraction decomposition process

Cases for partial fraction decomposition (types of factors in the denominator)

Case I: Linear factor

$$\frac{\text{stuff}}{x+4} \to \frac{A}{x+4}$$

Case II: Repeated linear factor

$$\frac{\text{stuff}}{(x+4)^3} \to \frac{A}{x+4} + \frac{B}{(x+4)^2} + \frac{C}{(x+4)^3}$$

Case III: Irreducible (prime) quadratic factor

$$\frac{\text{stuff}}{x^2+4} \to \frac{Ax+B}{x^2+4}$$

Case IV: Repeated irreducible quadratic factor

$$\frac{\text{stuff}}{\left(x^2+4\right)^3} \to \frac{Ax+B}{x^2+4} + \frac{Cx+D}{\left(x^2+4\right)^2} + \frac{Ex+F}{\left(x^2+4\right)^3}$$

Exercises:

Evaluate the integral.

7.4b

$$\int \frac{3t-2}{t+1}\,dt$$

$$\int_0^1 \frac{x-4}{x^2-5x+6}\,dx$$

7.4c

$$\int_{2}^{3} \frac{x(3-5x)}{(3x-1)(x-1)^2}\,dx$$

7.4d

$$\int_0^1 \frac{x}{x^2 + 4x + 13}\, dx$$

7.4e

Make a substitution to express the integrand as a rational function and then evaluate the integral.

$$\int \frac{dx}{2\sqrt{x+3}+x}$$