

7.4 - Integration of Rational Functions by Partial Fractions

7.4a

Start with a proper rational function. If the function is an improper rational function, use long division before starting the partial fraction decomposition process

[illegible]

Cases for partial fraction decomposition (types of factors in the denominator)

Case I: Linear factor

$$\frac{\text{stuff}}{x + 4} \rightarrow \frac{A}{x + 4}$$

Case II: Repeated linear factor

$$\frac{\text{stuff}}{(x + 4)^3} \rightarrow \frac{A}{x + 4} + \frac{B}{(x + 4)^2} + \frac{C}{(x + 4)^3}$$

Case III: Irreducible (prime) quadratic factor

$$\frac{\text{stuff}}{x^2 + 4} \rightarrow \frac{Ax + B}{x^2 + 4}$$

Case IV: Repeated irreducible quadratic factor

$$\frac{\text{stuff}}{(x^2 + 4)^3} \rightarrow \frac{Ax + B}{x^2 + 4} + \frac{Cx + D}{(x^2 + 4)^2} + \frac{Ex + F}{(x^2 + 4)^3}$$

7.4b

Exercises:

Evaluate the integral.

$$\int \frac{3t - 2}{t + 1} dt$$

$$\int_0^1 \frac{x - 4}{x^2 - 5x + 6} dx$$

7.4c

$$\int_2^3 \frac{x(3-5x)}{(3x-1)(x-1)^2} dx$$

7.4d

$$\int_0^1 \frac{x}{x^2 + 4x + 13} dx$$

7.4e

Make a substitution to express the integrand as a rational function and then evaluate the integral.

$$\int \frac{dx}{2\sqrt{x+3} + x}$$
