

7.2 - Trigonometric Integrals

7.2a

Exercise:

Evaluate the integral.

$$\int \sin^3 \theta \cos^4 \theta d\theta$$

Pythagorean Identities

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\tan^2 \theta + 1 = \sec^2 \theta$$

$$1 + \cot^2 \theta = \csc^2 \theta$$

7.2b

Exercise:

Evaluate the integral.

$$\int_0^{\pi} \sin^2 t \cos^4 t \, dt$$

Power-reducing formulas

$$\cos^2 \theta = \frac{1}{2} (1 + \cos 2\theta)$$

$$\sin^2 \theta = \frac{1}{2} (1 - \cos 2\theta)$$

The double-angle formula

$$\sin 2\theta = 2 \sin \theta \cos \theta$$

7.2c

Exercises:

Evaluate the integral.

$$\int \frac{\sin^2\left(\frac{1}{t}\right)}{t^2} dt$$

$$\int \tan^2 x \cos^3 x \, dx$$

7.2d

$$\int x \sin^3 x \, dx$$

7.2e

$$\int_0^{\frac{\pi}{4}} \sec^6 x \tan^6 x \, dx$$

$$\int \tan^5 x \sec^3 x \, dx$$

7.2f

Useful strategies

For $\int \sin^m \theta \cos^n \theta d\theta$:

Odd power
that's du

Even powers
power-reducing formula

For $\int \tan^m \theta \sec^n \theta d\theta$

Even power on secant
use $\sec^2 \theta = du$

Odd power on tangent

use $\sec \theta \tan \theta = du$

Two useful integration rules

$$\int \tan x \, dx = \ln |\sec x| + C$$

$$\int \sec x \, dx = \ln |\sec x + \tan x| + C$$
