6.8 - Indeterminate Forms and l'Hospital's Rule

6.8a

l'Hôspital's Rule for indeterminate forms

Suppose that f and g are differentiable and $g'(x) \neq 0$ on an open interval I that contains a (except possibly at a). Suppose that

$\lim_{x \to a} f(x) = 0$ and $\lim_{x \to a} = g(x) = 0$
or that $\lim_{x \to a} f(x) = \pm\infty$ and $\lim_{x \to a} = g(x) = \pm\infty$

Then $\lim_{x \to a} \dfrac{f(x)}{g(x)} = \lim_{x \to a} \dfrac{f'(x)}{g'(x)}$ if the limit on the right-hand side exists or is ∞ or $-\infty$

Indeterminate forms

$\frac{0}{0}, \frac{\infty}{\infty}, 0 \cdot \infty, \infty - \infty, 0^0, \infty^0$, and 1^∞

Exercises:

Find the limit. Use l'Hôspital's Rule where appropriate. If there is a more elementary method, consider using it. If l'Hôspital's Rule doesn't apply, explain why.

6.8b

$$\lim_{x \to 0} \frac{x^2}{1 - \cos x}$$

$$\lim_{\theta \to \pi} \frac{1 + \cos \theta}{1 - \cos \theta}$$

$$\lim_{x \to \infty} \frac{\ln \sqrt{x}}{x^2}$$

$$\lim_{x \to 0} \frac{\sinh x - x}{x^3}$$

6.8c

$$\lim_{t\to 0} \frac{8^t - 5^t}{t}$$

$$\lim_{x\to a^+} \frac{\cos x \ln(x-a)}{\ln\left(e^x - e^a\right)}$$

6.8d

$$\lim_{x \to -\infty} x \ln\left(1 - \frac{1}{x}\right)$$

$$\lim_{x \to 1^+} \left[\ln\left(x^7 - 1\right) - \ln\left(x^5 - 1\right)\right]$$

6.8e

$$\lim_{x \to 0^+} (\tan 2x)^x$$

6.8f

$$\lim_{x \to \infty} \left(1 + \frac{a}{x}\right)^{bx}$$

6.8g

Proof:

l'Hôspital's Rule

Suppose that f and g are differentiable and $g'(x) \neq 0$ on an open interval I that contains a (except possibly at a). Suppose that

$\lim_{x \to a} f(x) = 0$ and $\lim_{x \to a} = g(x) = 0$
or that $\lim_{x \to a} f(x) = \pm\infty$ and $\lim_{x \to a} = g(x) = \pm\infty$

Then $\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$ if the limit on the right-hand side exists or is ∞ or $-\infty$

Indeterminate forms

$\frac{0}{0}, \frac{\infty}{\infty}, 0 \cdot \infty, \infty - \infty, 0^0, \infty^0$, and 1^∞