6.6 - Inverse Trigonometric Functions

6.6a

A brief review of inverse trigonometric functions

6.6b

Calculus of inverse trigonometric functions

$$\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}} \Longrightarrow \int \frac{1}{\sqrt{1-x^2}}\, dx = \sin^{-1} x + C$$

$$\frac{d}{dx}(\cos^{-1} x) = -\frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\left(\tan^{-1} x\right) = \frac{1}{x^2+1} \Longrightarrow \int \frac{1}{x^2+1}\, dx = \tan^{-1} x + C$$

$$\frac{d}{dx}\left(\cot^{-1} x\right) = -\frac{1}{x^2+1}$$

$$\frac{d}{dx}\left(\sec^{-1} x\right) = \frac{1}{x\sqrt{x^2-1}} \Longrightarrow \int \frac{1}{x\sqrt{x^2-1}}\, dx = \sec^{-1} x + C$$

$$\frac{d}{dx}\left(\csc^{-1} x\right) = -\frac{1}{x\sqrt{x^2-1}}$$

Note: the formula for the derivative of the inverse secant assumes a restricted domain for the secant of $\{x|\ 0 < x < \frac{\pi}{2}\ or\ \pi < x < \frac{3\pi}{2}\}$. If instead the restricted domain is $\left(0,\ \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2},\ \pi\right)$, then the differentiation rule becomes $\frac{d}{dx}\left(\sec^{-1} x\right) = \frac{1}{|x|\ \sqrt{x^2-1}}$.

6.6c

Exercises:

Find the derivative of the function. Simplify where possible.

$g(x) = \arccos \sqrt{x}$

$y = \cos^{-1}\left(\sin^{-1} t\right)$

$R(t) = \arcsin\left(\frac{1}{t}\right)$

6.6d

$y = \arctan\sqrt{\dfrac{1-x}{1+x}}$

6.6e

Evaluate the integral

$$\int_{-\frac{1}{\sqrt{2}}}^{\frac{1}{\sqrt{2}}} \frac{6}{\sqrt{1-p^2}}\, dp$$

$$\int_0^{\frac{\sqrt{3}}{4}} \frac{dx}{1+16x^2}$$