

6.6 - Inverse Trigonometric Functions

6.6a

A brief review of inverse trigonometric functions

[illegible]

This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

6.6b

Calculus of inverse trigonometric functions

$$\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}} \Rightarrow \int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1} x + C$$

$$\frac{d}{dx}(\cos^{-1} x) = -\frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\tan^{-1} x) = \frac{1}{x^2+1} \Rightarrow \int \frac{1}{x^2+1} dx = \tan^{-1} x + C$$

$$\frac{d}{dx}(\cot^{-1} x) = -\frac{1}{x^2+1}$$

$$\frac{d}{dx}(\sec^{-1} x) = \frac{1}{x\sqrt{x^2-1}} \Rightarrow \int \frac{1}{x\sqrt{x^2-1}} dx = \sec^{-1} x + C$$

$$\frac{d}{dx}(\csc^{-1} x) = -\frac{1}{x\sqrt{x^2-1}}$$

Note: the formula for the derivative of the inverse secant assumes a restricted domain for the secant of $\{x \mid 0 < x < \frac{\pi}{2} \text{ or } \pi < x < \frac{3\pi}{2}\}$. If instead the restricted domain is

$\left(0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right)$, then the differentiation rule becomes $\frac{d}{dx}(\sec^{-1} x) = \frac{1}{|x|\sqrt{x^2-1}}$.

6.6c

Exercises:

Find the derivative of the function. Simplify where possible.

$$g(x) = \arccos \sqrt{x}$$

$$y = \cos^{-1}(\sin^{-1} t)$$

$$R(t) = \arcsin\left(\frac{1}{t}\right)$$

6.6d

$$y = \arctan \sqrt{\frac{1-x}{1+x}}$$

6.6e

Evaluate the integral

$$\int_{-1/\sqrt{2}}^{1/\sqrt{2}} \frac{6}{\sqrt{1-p^2}} dp$$

$$\int_0^{\sqrt{3}/4} \frac{dx}{1+16x^2}$$
