

## 6.4 - Derivatives of Logarithmic Functions

### 6.4a

Calculus of the natural logarithmic function

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$$\frac{d}{dx} [\ln |x|] = \frac{1}{x}, x \neq 0$$

$$\int \frac{1}{x} dx = \ln |x| + C$$

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**Exercise:**

Show that  $\int \tan x \, dx = \ln |\sec x| + C$ .

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## 6.4b

### Exercises:

Differentiate the function

$$f(x) = \ln(\sin^2 x)$$

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$$h(x) = \ln(x + \sqrt{x^2 - 1})$$

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$$y = \ln(\csc x - \cot x)$$

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## 6.4c

### Exercises:

Evaluate the integral.

$$\int_0^3 \frac{dx}{5x + 1}$$

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$$\int \frac{\cos x}{2 + \sin x} dx$$

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## 6.4d

Calculus of logs and exponentials with bases other than  $e$

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$$\frac{d}{dx} (\log_b x) = \frac{1}{x \ln b}$$

$$\frac{d}{dx}(b^x) = \ln b \cdot b^x$$

$$\int b^x dx = \frac{b^x}{\ln b} + C$$

### 6.4e

Definition: (Euler's number  $e$ )

[illegible]

$$e = \lim_{x \rightarrow 0} (1 + x)^{\frac{1}{x}}$$

$$e = \lim_{n \rightarrow \infty} \left( 1 + \frac{1}{n} \right)^n$$

## 6.4f

### Exercise:

Use logarithmic differentiation to find the derivative of the function.

$$y = \frac{e^{-x} \cos^2 x}{x^2 + x + 1}$$

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## 6.4g

### Exercise:

Use logarithmic differentiation to find the derivative of the function.

$$y = x^{\cos x}$$

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## Process for logarithmic differentiation

Take the natural log of both sides

Use properties of logs

Differentiate (usually with respect to  $x$ )—use implicit differentiation for  $y$

Give  $\frac{dy}{dx}$  or  $y'$  in terms of  $x$  (or whatever the independent variable is)

## Summary of techniques involving exponents

(variable base)<sup>constant exponent</sup> –power rule (ex:  $y = x^2$ )

(constant base)<sup>variable exponent</sup> –rules for exponentials (ex:  $y = e^x$  or  $y = 2^x$ )

(variable base)<sup>variable exponent</sup> –logarithmic differentiation

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