

11.9 - Representations of Functions as Power Series

11.9a

Exercises:

Find a power series representation for $f(x) = \frac{1}{1-x}$.

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + \dots$$

Express $\frac{1}{1+x^2}$ as a power series.

11.9b

Exercises:

Find a power series representation for the function and determine the interval of convergence.

$$f(x) = \frac{5}{1 - 4x^2}$$

$$f(x) = \frac{4}{2x + 3}$$

$$f(x) = \frac{x}{2x^2 + 1}$$

11.9c

More power series representations using term-by-term integration

$$\ln(1+x) = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n} x^n$$

$$\tan^{-1} x = \sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1} x^{2n+1}$$

11.9d

Exercises:

Find a power series representation for the function and determine the interval of convergence.

$$f(x) = x^2 \tan^{-1}(x^3)$$

$$f(x) = \left(\frac{x}{2-x} \right)^3$$

11.9e

Exercise:

Use a power series to approximate the definite integral to six decimal places.

$$\int_0^{0.2} x \ln(1 + x^2) \, dx$$
