11.6 - Absolute Convergence and the Ratio and Root Tests

11.6a

The Ratio Test:

i. If $\lim_{n\to\infty} |\frac{a_{n+1}}{a_n}| = L < 1$, then the series $\sum a_n$ is convergent

ii. If $\lim_{n\to\infty} |\frac{a_{n+1}}{a_n}| = L > 1$ or is ∞, then the series $\sum a_n$ is divergent

iii. If $\lim_{n\to\infty} |\frac{a_{n+1}}{a_n}| = 1$, the Ratio Test is inconclusive

Exercises:

Use the Ratio Test to determine whether the series is convergent or divergent.

11.6b

$$\sum_{n=1}^{\infty} \frac{(-2)^n}{n^2}$$

$$\sum_{n=1}^{\infty} \frac{n^{10}}{(-10)^{n+1}}$$

11.6c

$$\sum_{n=1}^{\infty} \frac{n!}{n^n}$$

11.6d

$$\frac{2}{3} + \frac{2 \cdot 5}{3 \cdot 5} + \frac{2 \cdot 5 \cdot 8}{3 \cdot 5 \cdot 7} + \frac{2 \cdot 5 \cdot 8 \cdot 11}{3 \cdot 5 \cdot 7 \cdot 9} + \cdots$$

11.6e

The Root Test:

i. If $\lim_{n\to\infty} \sqrt[n]{|a_n|} = L < 1$, then the series $\sum a_n$ is convergent

ii. If $\lim_{n\to\infty} \sqrt[n]{|a_n|} = L > 1$ or is ∞, then the series $\sum a_n$ is divergent

iii. If $\lim_{n\to\infty} \sqrt[n]{|a_n|} = 1$, the Root Test is inconclusive

Exercises:

Use the Root Test to determine whether the series is convergent or divergent.

$$\sum_{n=1}^{\infty} \frac{(-2)^n}{n^n}$$

$$\sum_{n=1}^{\infty} \left(-\frac{2n}{n+1}\right)^{5n}$$

11.6f

Exercises:

Use any test to determine whether the series is absolutely convergent, conditionally convergent, or divergent.

$$\sum_{n=1}^{\infty} \left(\frac{1-n}{2+3n} \right)^n$$

$$\sum_{n=1}^{\infty} \frac{\sin\left(\frac{n\pi}{6}\right)}{1+n\sqrt{n}}$$