

11.6 - Absolute Convergence and the Ratio and Root Tests

11.6a

The Ratio Test:

i. If $\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = L < 1$, then the series $\sum a_n$ is convergent

ii. If $\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = L > 1$ or is ∞ , then the series $\sum a_n$ is divergent

iii. If $\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = 1$, the Ratio Test is inconclusive

Exercises:

Use the Ratio Test to determine whether the series is convergent or divergent.

11.6b

$$\sum_{n=1}^{\infty} \frac{(-2)^n}{n^2}$$

$$\sum_{n=1}^{\infty} \frac{n^{10}}{(-10)^{n+1}}$$

11.6c

$$\sum_{n=1}^{\infty} \frac{n!}{n^n}$$

11.6d

$$\frac{2}{3} + \frac{2 \cdot 5}{3 \cdot 5} + \frac{2 \cdot 5 \cdot 8}{3 \cdot 5 \cdot 7} + \frac{2 \cdot 5 \cdot 8 \cdot 11}{3 \cdot 5 \cdot 7 \cdot 9} + \dots$$

This image shows a blank sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

11.6e

The Root Test:

i. If $\lim_{n \rightarrow \infty} \sqrt[n]{|a_n|} = L < 1$, then the series $\sum a_n$ is convergent

ii. If $\lim_{n \rightarrow \infty} \sqrt[n]{|a_n|} = L > 1$ or is ∞ , then the series $\sum a_n$ is divergent

iii. If $\lim_{n \rightarrow \infty} \sqrt[n]{|a_n|} = 1$, the Root Test is inconclusive

Exercises:

Use the Root Test to determine whether the series is convergent or divergent.

$$\sum_{n=1}^{\infty} \frac{(-2)^n}{n^n}$$

$$\sum_{n=1}^{\infty} \left(-\frac{2n}{n+1} \right)^{5n}$$

11.6f

Exercises:

Use any test to determine whether the series is absolutely convergent, conditionally convergent, or divergent.

$$\sum_{n=1}^{\infty} \left(\frac{1-n}{2+3n} \right)^n$$

$$\sum_{n=1}^{\infty} \frac{\sin\left(\frac{n\pi}{6}\right)}{1+n\sqrt{n}}$$
