

## 11.5 - Alternating Series

## 11.5a

### The Alternating Series Test (statement and proof):

If the alternating series

$$\sum_{n=1}^{\infty} (-1)^{n-1} b_n = b_1 - b_2 + b_3 - b_4 + \cdots, \text{ where } b_n > 0, \text{ satisfies}$$

i.  $b_{n+1} \leq b_n$  for all  $n$

$$\text{ii. } \lim_{n \rightarrow \infty} b_n = 0$$

then the series is convergent.

[illegible]

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## 11.5b

## Exercises:

Test the series for convergence or divergence.

$$\sum_{n=0}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n+1}}$$

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$$\sum_{n=1}^{\infty} (-1)^{n+1} n e^{-n}$$

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$$\sum_{n=1}^{\infty} (-1)^{n-1} \arctan n$$

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**11.5c**

### Theorem (statement and proof):

The Alternating Series Estimation Theorem: If  $s = \sum_{n=1}^{\infty} (-1)^{n-1} b_n$  is a series that converges by the Alternating Series Test, then  $|R_n| \leq b_{n+1}$ .

[illegible]

**Exercise:**

Use the Alternating Series Estimation Theorem to estimate the sum correct to four decimal places.

$$\sum_{n=1}^{\infty} (-1)^{n-1} \frac{n}{8^n}$$

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**11.5d**

Definitions: (absolute and conditional convergence)

A series  $\sum a_n$  is called absolutely convergent if the series of absolute values  $\sum |a_n|$  is convergent.

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A series  $\sum a_n$  is conditionally convergent if it is convergent but the series of absolute values  $\sum |a_n|$  is divergent.

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**Theorem (statement and proof):**

If a series  $\sum a_n$  is absolutely convergent, then it is convergent.

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