

11.4 - The Comparison Tests

11.4a

The Comparison Test:

Suppose that $\sum a_n$ and $\sum b_n$ are series with positive terms.

- i. If $\sum b_n$ is convergent and $a_n \leq b_n$ for all n , then $\sum a_n$ is also convergent.
- ii. If $\sum b_n$ is divergent and $b_n \leq a_n$ for all n , then $\sum a_n$ is also divergent.

[illegible]

Exercise:

Determine whether the series converges or diverges.

$$\sum_{n=2}^{\infty} \frac{1}{\sqrt{n} - 1}$$

11.4b

The Limit Comparison Test:

Suppose that $\sum a_n$ and $\sum b_n$ are series with positive terms. If $\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = c$, where c is a positive, finite number, then either both series converge or both series diverge.

Two special cases:

If $\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = 0$ AND if $\sum b_n$ converges, then $\sum a_n$ also converges.

If $\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = \infty$ AND if $\sum b_n$ diverges, then $\sum a_n$ also diverges.

Exercises:

Determine whether the series converges or diverges.

11.4c

$$\sum_{n=3}^{\infty} \frac{n+2}{(n+1)^3}$$

11.4d

$$\sum_{n=1}^{\infty} \frac{6^n}{5^n - 1}$$

$$\sum_{k=1}^{\infty} \frac{k \sin^2 k}{1 + k^3}$$

$$\sum_{n=1}^{\infty} \frac{2}{\sqrt{n} + 2}$$
