

11.2 - Series

11.2a

Introduction to series

Partial sums

$$s_n = a_1 + a_2 + a_3 + \cdots + a_n$$

Sequence of partial sums

$$\{s_n\} = s_1, s_2, s_3, \dots, s_n$$

Convergence of a series

Definition: If the sequence $\{s_n\}$ is convergent and the limit $\lim_{n \rightarrow \infty} s_n = s$ exists as

a finite real number then the associated series $\sum a_n$ is **convergent**, and the sum

of the series is $\sum_{n=1}^{\infty} a_n = s$.

Convergence of a geometric series

$\sum_{n=1}^{\infty} a r^{n-1}$ is convergent if $|r| < 1$ and divergent otherwise. If $|r| < 1$, then $S = \frac{a}{1-r}$.

11.2b

Theorem (statement and proof):

If $\sum_{n=1}^{\infty} a_n$ is convergent, then $\lim_{n \rightarrow \infty} a_n = 0$.

Divergence Test:

If $\lim_{n \rightarrow \infty} a_n$ does not exist or if $\lim_{n \rightarrow \infty} a_n \neq 0$, then the series $\sum_{n=1}^{\infty} a_n$ is divergent.

11.2c

Exercises:

Determine whether the series is convergent or divergent. If it is convergent, find its sum.

$$\frac{1}{3} + \frac{2}{9} + \frac{1}{27} + \frac{1}{243} + \frac{2}{729} + \dots$$

Determine whether the series is convergent or divergent by expressing s_n as a telescoping sum. If it is convergent, find its sum.

$$\sum_{n=1}^{\infty} \ln \frac{n}{n+1}$$

11.2d

Exercises:

Find the values of x for which the series converges. Find the sum of the series for those values of x .

$$\sum_{n=1}^{\infty} (x + 2)^n$$

Express the number as a ratio of integers.

$$10.1\overline{35} = 10.135353535 \dots$$

A patient is injected with a drug every 12 hours. Immediately before each injection the concentration of the drug has been reduced by 90% and the new dose increases the concentration by 1.5 mg/L.

What is the concentration after three doses?

If C_n is the concentration after the n th dose, find a formula for C_n as a function of n .

What is the limiting value of the concentration?