

## 11.2 - Series

### 11.2a

## Introduction to series

[illegible]

## Partial sums

---

---

---

---

---

$$s_n = a_1 + a_2 + a_3 + \cdots + a_n$$

Sequence of partial sums

$$\{s_n\} = s_1, s_2, s_3, \dots, s_n, \dots$$

Convergence of a series

**Definition:** If the sequence  $\{s_n\}$  is convergent and the limit  $\lim_{n \rightarrow \infty} s_n = s$  exists as a finite real number then the associated series  $\sum a_n$  is **convergent**, and the sum

of the series is  $\sum_{n=1}^{\infty} a_n = s$ .

Convergence of a geometric series

---

---

---

---

---

---

---

---

$\sum_{n=1}^{\infty} ar^{n-1}$  is convergent if  $|r| < 1$  and divergent otherwise. If  $|r| < 1$ , then

$$S = \frac{a}{1 - r}.$$

---

---

---

---

---

---

---

## 11.2b

**Theorem (statement and proof):**

If  $\sum_{n=1}^{\infty} a_n$  is convergent, then  $\lim_{n \rightarrow \infty} a_n = 0$ .

---

---

---

---

---

---

---

---

---

Divergence Test:

If  $\lim_{n \rightarrow \infty} a_n$  does not exist or if  $\lim_{n \rightarrow \infty} a_n \neq 0$ , then the series  $\sum_{n=1}^{\infty} a_n$  is divergent.

## 11.2c

### Exercises:

Determine whether the series is convergent or divergent. If it is convergent, find its sum.

$$\frac{1}{3} + \frac{2}{9} + \frac{1}{27} + \frac{2}{81} + \frac{1}{243} + \frac{2}{729} + \dots$$

---

---

---

---

---

---

---

---

Determine whether the series is convergent or divergent by expressing  $s_n$  as a telescoping sum. If it is convergent, find its sum.

$$\sum_{n=1}^{\infty} \ln \frac{n}{n+1}$$

---

---

---

---

---

---

---

---

---

## 11.2d

### Exercises:

Find the values of  $x$  for which the series converges. Find the sum of the series for those values of  $x$ .

$$\sum_{n=1}^{\infty} (x+2)^n$$

---

---

---

---

---

---

---

---

---

---

Express the number as a ratio of integers.

$$10.\overline{135} = 10.135135135135 \dots$$

[illegible]

A patient is injected with a drug every 12 hours. Immediately before each injection the concentration of the drug has been reduced by 90% and the new dose increases the concentration by 1.5 mg/L.

What is the concentration after three doses?

---

---

---

If  $C_n$  is the concentration after the  $n$ th dose, find a formula for  $C_n$  as a function of  $n$ .

---

---

---

What is the limiting value of the concentration?

---

---