11.1 - Sequences

11.1a

Exercise:

Calculate, to four decimal places, the first ten terms of the sequence and use them to plot the graph of the sequence by hand. Does the sequence appear to have a limit? If so, calculate it. If not, explain why.

$$a_n = 2 + \frac{(-1)^n}{n}$$

Theorem (statement):

If $\lim_{n \to \infty} |a_n| = 0$, then $\lim_{n \to \infty} a_n = 0$.

Definition: A sequence $\{a_n\}$ has the **limit** L and we write $\lim_{n\to\infty} = L$ or $a_n \to L$ as $n \to \infty$ if for every $\varepsilon > 0$ there is a corresponding integer N such that if $n > N$, then $|a_n - L| < \varepsilon$.

Exercise:

$$a_n = 1 + \frac{10^n}{9^n}$$

11.1b

Exercises:

Determine whether the sequence converges or diverges. If it converges, find the limit.

$$a_n = \frac{3 + 5n^2}{1 + n}$$

$$a_n = \frac{4^n}{1 + 9^n}$$

$$a_n = e^{\frac{2n}{n+2}}$$

Theorems (statement):

If $\lim\limits_{x\to\infty} f(x) = L$ and $f(n) = a_n$ when n is an integer, then $\lim\limits_{x\to\infty} a_n = L$.

If $\lim\limits_{x\to\infty} a_n = L$ and the function f is continuous at L, then $\lim\limits_{x\to\infty} f(a_n) = f(L)$.

11.1c

Exercises:

Determine whether the sequence converges or diverges. If it converges, find the limit.

$$\left\{ \frac{\ln n}{\ln(2n)} \right\}$$

$$a_n = \frac{(-3)^n}{n!}$$

11.1d

Determine whether the sequence is increasing, decreasing, or not monotonic. Is the sequence bounded?

$$a_n = \frac{1-n}{2+n}$$

Theorem (statement):

Monotonic Sequence Theorem: Every bounded, monotonic sequence is convergent.

11.1e

Exercise:

Show that the sequence defined by

$$a_1 = 2,\ a_{n+1} = \frac{1}{3 - a_n}$$

satisfies $0 \leq a_n \leq 2$ and is decreasing. Deduce that the sequence is convergent and find its limit.