

10.4 - Areas and Lengths in Polar Coordinates

10.4a

Area in polar coordinates

$$A = \int_a^b \frac{1}{2} [f(\theta)]^2 d\theta$$

Between two curves f and g ($f \geq g$): $A = \frac{1}{2} \int_a^b \{[f(\theta)]^2 - [g(\theta)]^2\} d\theta$

Arc length

$$L = \int_a^b \sqrt{\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2} d\theta = \int_a^b \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} d\theta$$

10.4b

Exercises:

Graph the curve and find the area that it encloses.

$$r = 3 - 2 \cos 4\theta$$

10.4c

Find the area of the region that lies inside the first curve and outside the second curve.

$$r = 3 \sin \theta, r = 2 - \sin \theta$$

10.4d

Find the area of the region that lies inside both curves.

$$r = \sin 2\theta, r = \cos 2\theta$$

10.4e

Find the length of the polar curve.

$$r = 5^\theta, 0 \leq \theta \leq 2\pi$$

10.4f

Tangents to a polar curve: if $r = f(\theta)$ and $x = r \cos \theta$, $y = r \sin \theta$, then

$$\frac{dy}{d\theta} = \frac{dr}{d\theta} \sin \theta + r \cos \theta \text{ (by the product rule)}$$

$$\frac{dx}{d\theta} = \frac{dr}{d\theta} \cos \theta - r \sin \theta \text{ (product rule again)}$$

$$\frac{dy}{dx} = \frac{\frac{dr}{d\theta} \sin \theta + r \cos \theta}{\frac{dr}{d\theta} \cos \theta - r \sin \theta}$$

Horizontal tangents occur when $\frac{dy}{d\theta} = 0$ (if $\frac{dy}{dx} \neq 0$)

Vertical tangents occur when $\frac{dx}{d\theta} = 0$ (if $\frac{dx}{dy} \neq 0$)

Note: if both $\frac{dy}{d\theta} = 0$ and $\frac{dx}{d\theta} = 0$, we use l'Hospital's Rule

Exercises:

Find the slope of the tangent line to the given polar curve at the point specified by the value of θ .

$$r = 2 + \sin 3\theta, \theta = \frac{\pi}{4}$$

[illegible]

10.4g

Find the points on the given curve where the tangent line is horizontal or vertical.

$$r = 1 - \sin \theta$$


