10.3 - Polar Coordinates

10.3a

Rectangular-polar conversions

$x = r\cos\theta,\ y = r\sin\theta,\ x^2 + y^2 = r^2, \tan\theta = \dfrac{y}{x}$

10.3b

Exercises:

Identify the curve by finding a Cartesian equation for the curve.

$r = 4 \sec \theta$

$r^2 \sin 2\theta = 1$

Find a polar equation for the curve represented by the given Cartesian equation.

$4y^2 = x$

10.3c

Common types of polar equations

$r = a \pm b \sin\theta \, (or \cos\theta)$

- Cardioid if $a = b$
- Dimpled limaçon if $a > b$
- Limaçon with an inner loop if $a < b$

$r = a \sin n\theta \ \ (or \cos n\theta)$

- Rose with n petals if n is odd
- Rose with $2n$ petals if n is even
- Circle if $n = 1$

$r^2 = a^2 \cos 2\theta$ (lemniscate)

10.3d

Testing for symmetry in the polar plane

- With respect to the polar axis: replace $\theta \to -\theta$
- With respect to the line $\theta = \pi/2$: replace $\theta \to \pi - \theta$
- With respect to the pole: replace $r \to -r$ or $\theta \to \pi + \theta$

Note: Functions involving the sine function are typically symmetric with respect to the line $\theta = \pi/2$, and functions involving the cosine function are typically symmetric with respect to the polar axis.

10.3e

Exercise:

Sketch the curve with the given polar equation by first sketching the graph of r as a function of θ in Cartesian coordinates.

$r = 1 + 2\cos\theta$

10.3f

Exercise:

Use a graphing device to graph the polar curve. Choose the parameter interval to make sure that you produce the entire curve.

$r = 2 + \cos\left(\frac{9\theta}{4}\right)$