

10.2 - Calculus with Parametric Curves

10.2a

Calculus with curves defined by parametric equations $x = f(t)$, $y = g(t)$

Derivatives

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{g'(t)}{f'(t)}, \text{ if } \frac{dx}{dt} \neq 0$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx} \left(\frac{dy}{dx} \right) = \frac{\frac{d}{dt} \left(\frac{dy}{dx} \right)}{\frac{dx}{dt}} = \frac{\frac{d}{dt} \left(\frac{g'(t)}{f'(t)} \right)}{\frac{dx}{dt}}$$

Area

$$A = \int_a^b y \, dx = \int_\alpha^\beta g(t) f'(t) \, dt$$

Arc length

$$L = \int_{\alpha}^{\beta} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt = \int_{\alpha}^{\beta} \sqrt{[f'(t)]^2 + [g'(t)]^2} dt$$

Area of a surface of revolution

By rotating about the x -axis:
$$S = \int_{\alpha}^{\beta} 2\pi y \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt$$

By rotating about the y -axis:
$$S = \int_{\alpha}^{\beta} 2\pi x \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt$$

10.2b

Exercises:

Find an equation of the tangent to the curve at the point corresponding to the given value of the parameter.

$$x = e^t \sin \pi t, y = e^{2t}, t = 0$$

10.2c

Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. For which values of t is the curve concave upward?

$$x = t^2 + 1, y = e^t + 1$$

10.2d

Find points on the curve where the tangent is horizontal or vertical.

$$x = t^3 - 3t, y = t^3 - 3t^2$$

10.2e

Find the area of the region enclosed by the astroid $x = a \cos^3 \theta$, $y = a \sin^3 \theta$. Use a graphing device to graph the curve for $0 \leq \theta \leq 2\pi$ and an a -value of your choosing.

Example: What about $x = a \cos \theta$, $y = a \sin \theta$, $0 \leq \theta \leq 2\pi$?

10.2f

Find the exact length of the curve.

$$x = 3 \cos t - \cos 3t, y = 3 \sin t - \sin 3t, 0 \leq t \leq \pi$$

[illegible]

Example: What about $x = a \cos \theta$, $y = a \sin \theta$, $0 \leq \theta \leq 2\pi$?

10.2g

Find the exact area of the surface obtained by rotating the given curve about the y -axis.

$$x = 3t^2, y = 2t^3, 0 \leq t \leq 5$$
