8.2 - Area of a Surface of Revolution

Preparation (Watch at Home)

8.2a

Area of a surface of revolution

About the x-axis: $S = \int 2\pi y \, ds$

About the y-axis: $S = \int 2\pi x \, ds$

$ds = \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx = \sqrt{1 + \left(\frac{dx}{dy}\right)^2} dy$, whichever is most convenient
Limits of integration: $a \leq x \leq b$ for $y = f(x)$; for $c \leq y \leq d$ for $x = g(y)$

8.2b

Exercise:

Find the exact area of the surface obtained by rotating the curve about the x-axis.

$y = \sqrt{1 + e^x}$, $0 \leq x \leq 1$

$$y = \frac{x^3}{6} + \frac{1}{2x}, \frac{1}{2} \leq x \leq 1$$

In-Class Discussion

8.2c

Exercise:

The given curve is rotated about the y-axis. Find the area of the resulting surface.

$x^{2/3} + y^{2/3} = 1, 0 \leq y \leq 1$

In-Class Practice

1. a) set up an integral for the area of the surface obtained by rotating the curve $y = \tan^{-1} x, 0 \leq x \leq 2$ about (i) the x-axis and (ii) the y-axis.

b) Use www.wolframalpha.com (http://www.wolframalpha.com) to evaluate the surface areas correct to four decimal places.

2. Find the exact area of the surface obtained by rotating the curve $x = 1 + 2y^2$, $1 \leq y \leq 2$ about the x-axis.

3. Find the exact area of the surface obtained by rotating the curve $y = \frac{1}{3}x^{3/2}$, $0 \leq x \leq 12$ about the y-axis.