8.1 - Arc Length

Preparation (Watch at Home)

8.1a

Arc length formula for $y = f(x)$, $a \leq x \leq b$

$$L = \int_a^b \sqrt{1 + \left[f'(x)\right]^2}\, dx = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

8.1c

Arc length function

$$s(x) = \int_a^x \sqrt{1 + \left[f'(t)\right]^2}\, dt$$

Two forms for ds

$$ds = \sqrt{1 + \left(\tfrac{dy}{dx}\right)^2} dx = \sqrt{1 + \left(\tfrac{dx}{dy}\right)^2}\, dy$$

Arc length formula for $x = g(y)$, $c \le y \le d$

$$L = \int_c^d \sqrt{1 + \left[g'(y)\right]^2}\, dy = \int_c^d \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$$

Exercises:

Find the exact length of the curve.

$x = \frac{y^4}{8} + \frac{1}{4y^2}, 1 \leq y \leq 2$

8.1d

$y = \ln(\cos x), 0 \leq x \leq \frac{\pi}{3}$

$y = 1 - e^{-x}, 0 \leq x \leq 2$

In-Class Discussion

8.1b

Exercises:

Set up an integral that represents the length of the curve. Then use your calculator to find the length correct to four decimal places.

$y = xe^{-x}, 0 \leq x \leq 2$

Find the exact length of the curve.

$36y^2 = \left(x^2 - 4\right)^3, 2 \leq x \leq 3, y \geq 0$

In-Class Practice

1. Find the exact length of the curve $y = 3 + \frac{1}{2}\cosh 2x$, $0 \leq x \leq 1$.

2. Find the length of the arc of the curve $x^2 = (y-4)^3$ from point $P(1,\ 5)$ to $Q(8,\ 8)$.

3. a) Find the arc length function for the curve $y = \ln(\sin x)$, $0 < x < \pi$, with starting point $\left(\frac{\pi}{2},\ 0\right)$.

b) Graph both the curve and its arc length function on the same screen. Why is the arc length function negative when x is less than $\pi/2$?