7.8 - Improper Integrals

Preparation (Watch at Home)

7.8a

Types of improper integrals

Infinite interval: $\int_{-\infty}^{a} f(x)\,dx = \lim_{t\to-\infty} \int_{t}^{a} f(x)dx$ or $\int_{a}^{\infty} f(x)\,dx = \lim_{t\to\infty} \int_{a}^{t} f(x)dx$

Infinite discontinuity: $\int_{a}^{b} f(x)\,dx$, where there is $c \in [a,\ b]$ such that $\lim_{x\to c} f(x) = \pm\infty$. In this case we have

$$\int_{a}^{b} f(x)\,dx = \lim_{t\to c^-} \int_{a}^{t} f(x)dx + \lim_{t\to c^+} \int_{t}^{b} f(x)dx$$

Exercises:

Determine whether each integral is convergent or divergent. Evaluate those that are convergent.

$$\int_1^\infty \frac{e^{-1/x}}{x^2}\,dx$$

$$\int_0^\infty \sin\theta e^{\cos\theta}\,d\theta$$

7.8b

Exercises:

Determine whether each integral is convergent or divergent. Evaluate those that are convergent.

$$\int_2^{\infty} \frac{dv}{v^2 + 2v - 3}$$

$$\int_0^{1} \frac{dx}{\sqrt{1 - x^2}}$$

In-Class Discussion

7.8c

Exercise:

For what values of p is the integral $\int_1^\infty \frac{1}{x^p}\,dx$ convergent?

7.8d

Theorem (statement):

Comparison Theorem: If $f(x) \geq g(x) \geq 0$ (both continuous) for $x \geq a$, then

a) If $\int_a^\infty f(x)\,dx$ is convergent, then so is $\int_a^\infty g(x)\,dx$

b) If $\int_a^\infty g(x)\,dx$ is divergent, then so is $\int_a^\infty f(x)\,dx$

Exercise:

Use the Comparison Theorem to determine whether the integral is convergent or divergent.

$$\int_1^\infty \frac{1+\sin^2 x}{\sqrt{x}}\,dx$$

In-Class Practice

Determine whether each integral is convergent or divergent. Evaluate those that are convergent.

1. $\int_{-\infty}^0 \frac{z}{z^4+4}\,dz$

2. $\int_{-2}^{3} \frac{1}{x^4}\, dx$

3. $\int_{0}^{1} r \ln r\, dr$

4. Use the Comparison Theorem to determine whether the integral is convergent or divergent.

$$\int_0^\infty \frac{\arctan x}{2 + e^x}\, dx$$