

7.7 - Approximate Integration

Preparation (Watch at Home)

7.7a

The Midpoint Rule:

$$\int_a^b f(x) dx \approx \Delta x [f(\bar{x}_1) + f(\bar{x}_2) + \cdots f(\bar{x}_n)], \text{ where } \Delta x = \frac{b-a}{n} \text{ and } \bar{x}_i = \frac{x_{i-1} + x_i}{2}, \text{ the midpoint of } [x_{i-1}, x_i]$$

$$\text{Error is } |E_M| \leq \frac{K(b-a)^3}{24n^2}, \text{ where } |f''(x)| \leq K \text{ for } x \in [a, b]$$

7.7b

The Trapezoid Rule:

$$\int_a^b f(x) dx \approx \frac{\Delta x}{2} [f(x_0) + 2f(x_1) + 2f(x_2) + \cdots 2f(x_{n-1}) + f(x_n)], \text{ where}$$
$$\Delta x = \frac{b-a}{n} \text{ and } x_i = a + i\Delta x$$

$$\text{Error is } |E_T| \leq \frac{K(b-a)^3}{12n^2}, \text{ where } |f''(x)| \leq K \text{ for } x \in [a, b]$$

Exercises:

7.7e

Simpson's Rule:

[illegible]

$$\int_a^b f(x) \, dx \approx \frac{\Delta x}{3} [f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \cdots + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)],$$

where n is even, $\Delta x = \frac{b-a}{n}$, and $x_i = a + i\Delta x$

Error is $|E_S| \leq \frac{K(b-a)^5}{180n^4}$, where $|f^{(4)}(x)| \leq K$ for $x \in [a, b]$

In-Class Discussion

7.7c

Use (a) the Trapezoidal Rule and (b) the Midpoint Rule to approximate the given integral with the specified value of n . (Round your answers to six decimal places.)

[illegible]



b) How large do we have to choose n so that the approximations T_n and M_n to the integral in part (a) are accurate to within 0.0001?

In-Class Practice

1. The following instructions pertain to the integral $I = \int_0^{2\pi} e^{\cos x} dx$. For this exercise, please use the following tools: WolframAlpha (<https://www.wolframalpha.com>) a graphing utility such as Desmos (<https://www.desmos.com/calculator>) or Geogebra (<https://www.geogebra.org/graphing>), and Excel (or another spreadsheet program).

a) Use a graph to get a good upper bound for $|f''(x)|$.

b) Use M_{10} to approximate I .

- c) Use part (a) to estimate the error in part (b).
- d) Use WolframAlpha to approximate I .
- e) How does the actual error compare with the error estimate in part (c)?
- f) Use a graph to get a good upper bound for $|f^{(4)}(x)|$.
- g) Use S_{10} to approximate I .
- h) Use part (f) to estimate the error in part (g).
- i) How does the actual error compare with the error estimate in part (h)?
- j) How large should n be to guarantee that the size of the error in using S_{10} is less than 0.0001?
