6.7 - Hyperbolic Functions

6.7a

Definitions: (hyperbolic functions)

$$\sinh x = \frac{e^x - e^{-x}}{2} \quad \mathrm{csch} x = \frac{1}{\sinh x}$$

$$\cosh x = \frac{e^x + e^{-x}}{2} \quad \mathrm{sech} x = \frac{1}{\cosh x}$$

$$\tanh x = \frac{\sinh x}{\cosh x} \quad \coth x = \frac{\cosh x}{\sinh x}$$

6.7b

Exercises:

Find the numerical value of each expression.
a) $\sinh 4$ b) $\sinh(\ln 4)$

Prove the identity.
$\cosh(x + y) = \cosh x \cosh y + \sinh x \sinh y$

6.7c

Derivatives of hyperbolic functions

$$\frac{d}{dx}(\sinh x) = \cosh x$$

$$\frac{d}{dx}(\cosh x) = \sinh x$$

$$\frac{d}{dx}(\tanh x) = \text{sech}^2 x$$

$$\frac{d}{dx}(sech x) = -\text{sech} x \tanh x$$

$$\frac{d}{dx}(\text{csch} x) = -\text{csch} x \coth x$$

$$\frac{d}{dx}(\coth x) = -\text{csch}^2 x$$

Note that reversing these gives integration rules

Exercises:

Find the derivative. Simplify where possible.

$F(t) = \ln(\sinh t)$

Evaluate the integral.

$$\int \frac{\operatorname{sech}^2 x}{2 + \tanh x}\, dx$$

6.7d

Inverse hyperbolic functions

$\sinh^{-1} x = \ln(x + \sqrt{x^2 + 1}),\ x \in \mathbb{R}$

$\cosh^{-1} x = \ln(x + \sqrt{x^2 - 1}), x \geq 1$

$\tanh^{-1} x = \dfrac{1}{2} \ln\left(\frac{1+x}{1-x}\right), -1 < x < 1$

Derivatives of inverse hyperbolic functions

$$\frac{d}{dx}\left(\sinh^{-1} x\right) = \frac{1}{\sqrt{1+x^2}} \qquad \frac{d}{dx}\left(\operatorname{csch}^{-1} x\right) = -\frac{1}{|x|\sqrt{x^2+1}}$$

$$\frac{d}{dx}\left(\cosh^{-1} x\right) = \frac{1}{\sqrt{x^2-1}} \qquad \frac{d}{dx}\left(\operatorname{sech}^{-1} x\right) = -\frac{1}{x\sqrt{1-x^2}}$$

$$\frac{d}{dx}\left(\tanh^{-1} x\right) = \frac{1}{1-x^2}, |x| < 1 \qquad \frac{d}{dx}\left(\coth^{-1} x\right) = \frac{1}{1-x^2}, |x| > 1$$

Exercise:

Evaluate the integral.

$$\int_0^1 \frac{1}{\sqrt{16t^2+1}}\, dt$$