

6.7 - Hyperbolic Functions

6.7a

Definitions: (hyperbolic functions)

$$\sinh x = \frac{e^x - e^{-x}}{2} \quad \operatorname{csch} x = \frac{1}{\sinh x}$$

$$\cosh x = \frac{e^x + e^{-x}}{2} \quad \operatorname{sech} x = \frac{1}{\cosh x}$$

$$\tanh x = \frac{\sinh x}{\cosh x} \quad \coth x = \frac{\cosh x}{\sinh x}$$

6.7b

Exercises:

Find the numerical value of each expression.

a) $\sinh 4$

b) $\sinh(\ln 4)$

Prove the identity.

$$\cosh(x + y) = \cosh x \cosh y + \sinh x \sinh y$$

6.7c

Derivatives of hyperbolic functions

$$\frac{d}{dx}(\sinh x) = \cosh x$$

$$\frac{d}{dx}(\cosh x) = \sinh x$$

$$\frac{d}{dx}(\tanh x) = \operatorname{sech}^2 x$$

$$\frac{d}{dx}(\operatorname{sech} x) = -\operatorname{sech} x \tanh x$$

$$\frac{d}{dx}(\operatorname{csch} x) = -\operatorname{csch} x \coth x$$

$$\frac{d}{dx}(\coth x) = -\operatorname{csch}^2 x$$

Note that reversing these gives integration rules

Exercises:

Find the derivative. Simplify where possible.

$$F(t) = \ln(\sinh t)$$

Evaluate the integral.

$$\int \frac{\operatorname{sech}^2 x}{2 + \tanh x} dx$$

6.7d

Inverse hyperbolic functions

$$\sinh^{-1} x = \ln(x + \sqrt{x^2 + 1}), x \in \mathbb{R}$$

$$\cosh^{-1} x = \ln(x + \sqrt{x^2 - 1}), x \geq 1$$

$$\tanh^{-1} x = \frac{1}{2} \ln\left(\frac{1+x}{1-x}\right), -1 < x < 1$$

Derivatives of inverse hyperbolic functions

$$\frac{d}{dx} (\sinh^{-1} x) = \frac{1}{\sqrt{1+x^2}} \quad \frac{d}{dx} (\operatorname{csch}^{-1} x) = -\frac{1}{|x|\sqrt{x^2+1}}$$

$$\frac{d}{dx} (\cosh^{-1} x) = \frac{1}{\sqrt{x^2-1}} \quad \frac{d}{dx} (\operatorname{sech}^{-1} x) = -\frac{1}{x\sqrt{1-x^2}}$$

$$\frac{d}{dx} (\tanh^{-1} x) = \frac{1}{1-x^2}, |x| < 1 \quad \frac{d}{dx} (\operatorname{coth}^{-1} x) = \frac{1}{1-x^2}, |x| > 1$$

Exercise:

Evaluate the integral.

$$\int_0^1 \frac{1}{\sqrt{16t^2 + 1}} dt$$
