

6.6 - Inverse Trigonometric Functions

Preparation (Watch at Home)

6.6a

A brief review of inverse trigonometric functions

[illegible]

[illegible]

6.6b

Calculus of inverse trigonometric functions

$$\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}} \implies \int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1} x + C$$

$$\frac{d}{dx}(\cos^{-1} x) = -\frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\tan^{-1} x) = \frac{1}{x^2+1} \implies \int \frac{1}{x^2+1} dx = \tan^{-1} x + C$$

$$\frac{d}{dx}(\cot^{-1} x) = -\frac{1}{x^2+1}$$

$$\frac{d}{dx}(\sec^{-1} x) = \frac{1}{x\sqrt{x^2-1}} \implies \int \frac{1}{x\sqrt{x^2-1}} dx = \sec^{-1} x + C$$

$$\frac{d}{dx}(\csc^{-1} x) = -\frac{1}{x\sqrt{x^2-1}}$$

Note: the formula for the derivative of the inverse secant assumes a restricted domain for the secant of $\{x \mid 0 < x < \frac{\pi}{2} \text{ or } \pi < x < \frac{3\pi}{2}\}$. If instead the restricted domain is

$\left(0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right)$, then the differentiation rule becomes $\frac{d}{dx}(\sec^{-1} x) = \frac{1}{|x|\sqrt{x^2-1}}$.

6.6c

Exercises:

Find the derivative of the function. Simplify where possible.

$$g(x) = \arccos \sqrt{x}$$

$$y = \cos^{-1} (\sin^{-1} t)$$

$$R(t) = \arcsin \left(\frac{1}{t} \right)$$

6.6e

Evaluate the integral

$$\int_{-\frac{1}{\sqrt{2}}}^{\frac{1}{\sqrt{2}}} \frac{6}{\sqrt{1-p^2}} dp$$

$$\int_0^{\frac{\sqrt{3}}{4}} \frac{dx}{1+16x^2}$$

In-Class Discussion

6.6d

$$y = \arctan \sqrt{\frac{1-x}{1+x}}$$

In-Class Practice

Find the derivative of the function. Simplify where possible.

$$y = \tan^{-1}(x^2)$$

2. Find the derivative of the function. Find the domains of the function and its derivative.

$$f(x) = \arcsin(e^x)$$

3. Find an equation of the tangent line to the curve $y = 3 \arccos\left(\frac{x}{2}\right)$ at the point $(1, \pi)$.

4. Find the limit.

$$\lim_{x \rightarrow \infty} \arccos \left(\frac{1 + x^2}{1 + 2x^2} \right)$$

Evaluate the integral.

5. $\int \frac{(\arctan x)^2}{x^2 + 1} dx$

6. $\int \frac{1}{x \sqrt{x^2 - 4}} dx$

7. $\int \frac{1}{25 + x^2} dx$
