

6.4 - Derivatives of Logarithmic Functions

Preparation (Watch at Home)

6.4a

Calculus of the natural logarithmic function

$$\frac{d}{dx} [\ln |x|] = \frac{1}{x}, x \neq 0$$

$$\int \frac{1}{x} dx = \ln |x| + C$$

Exercise:

Show that $\int \tan x \, dx = \ln |\sec x| + C$.

6.4b**Exercises:**

Differentiate the function

$$f(x) = \ln(\sin^2 x)$$

$$h(x) = \ln(x + \sqrt{x^2 - 1})$$

$$y = \ln(\csc x - \cot x)$$

6.4c

Exercises:

Evaluate the integral.

$$\int_0^3 \frac{dx}{5x + 1}$$

$$\int \frac{\cos x}{2 + \sin x} dx$$

6.4d

Calculus of logs and exponentials with bases other than e

$$\frac{d}{dx} (\log_b x) = \frac{1}{x \ln b}$$

$$\frac{d}{dx} (b^x) = \ln b \cdot b^x$$

$$\int b^x dx = \frac{b^x}{\ln b} + C$$

6.4f

Exercise:

Use logarithmic differentiation to find the derivative of the function.

$$y = \frac{e^{-x} \cos^2 x}{x^2 + x + 1}$$

In-Class Discussion

6.4e

Definition: (Euler's number e)

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$$e = \lim_{x \rightarrow 0} (1 + x)^{\frac{1}{x}}$$

$$e = \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n} \right)^n$$

6.4g

Exercise:

Use logarithmic differentiation to find the derivative of the function.

$$y = x^{\cos x}$$

Process for logarithmic differentiation

Take the natural log of both sides

Use properties of logs

Differentiate (usually with respect to x)—use implicit differentiation for y

Give $\frac{dy}{dx}$ or y' in terms of x (or whatever the independent variable is)

Summary of techniques involving exponents

(variable base)^{constant exponent} –power rule (ex: $y = x^2$)

(constant base)^{variable exponent} –rules for exponentials (ex: $y = e^x$ or $y = 2^x$)

(variable base)^{variable exponent} –logarithmic differentiation

In-Class Practice

Differentiate the function.

1. $g(t) = \sqrt{1 + \ln t}$

2. $g(x) = x \sin(2^x)$

3. $F(t) = 3^{\cos 2t}$

Use logarithmic differentiation to find the derivative of the function.

4. $y = \sqrt{x}e^{x^2-x}(x+1)^{\frac{2}{3}}$

5. $y = (\sin x)^{\ln x}$

Evaluate the integral.

6. $\int \frac{\cos(\ln t)}{t} dt$

$$7. \int \frac{e^x}{e^x + 1} dx$$
