

6.1 - Inverse Functions

Preparation (Watch at Home)

6.1a

Definition: (one-to-one)

A function f is one-to-one if $f(x_2) \neq f(x_1)$ whenever $x_2 \neq x_1$.

Description of inverse functions

This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

6.1b

Theorems (statement and proof):

If f is one-to-one and continuous on $[a, b]$, then f^{-1} is also continuous.

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$$(f^{-1})'(a) = \frac{1}{f'(f^{-1}(a))}$$

6.1c

Exercises:

If $f(x) = x^5 + x^3 + x$, find $f^{-1}(3)$ and $f(f^{-1}(2))$.

Find $(f^{-1})'(a)$ for $f(x) = x^3 + 3 \sin x + 2 \cos x$ and $a = 2$.

In-Class Discussion

6.1d

Exercise:

Let $f(x) = \sqrt{x-2}$ and $a = 2$.

- a) Show that f is one-to-one
-

Find $(f^{-1})'(a) = \frac{1}{f'(f^{-1}(a))}$

In-Class Practice

1. Let $f(x) = \frac{1}{x-1}$, $x > 1$, $a = 2$.

a) Show that f is one-to-one.

b) Use Theorem 7 to find $(f^{-1})'(a)$.

c) Calculate $f^{-1}(x)$ and state the domain and range of f^{-1} .

d) Calculate $(f^{-1})'(a)$ from the formula in part (c) and check that it agrees with the result of part (b).

e) Sketch the graphs of f and f^{-1} on the same axes.

[illegible]

2. Find $(f^{-1})'(a)$ for $f(x) = \sqrt{x^3 + 4x + 4}$, $a = 3$.

3. If g is an increasing function such that $g(2) = 8$ and $g'(2) = 5$, calculate $(g^{-1})'(8)$.

4. If f is a one-to-one, twice differentiable function with inverse function g , show that

$$g''(x) = -\frac{f''(g(x))}{[f'(g(x))]^3}$$

What conclusion can you draw about the graph of $f^{-1}(x)$ if f is increasing and concave upward?
