6.1 - Inverse Functions

Preparation (Watch at Home)

6.1a

Definition: (one-to-one)

A function f is one-to-one if $f(x_2) \neq f(x_1)$ whenever $x_2 \neq x_1$.

Description of inverse functions

6.1b

Theorems (statement and proof):

If f is one-to-one and continuous on $[a,\ b]$, then f^{-1} is also continuous.

$$\left(f^{-1}\right)'(a) = \frac{1}{f'\left(f^{-1}(a)\right)}$$

6.1c

Exercises:

If $f(x) = x^5 + x^3 + x$, find $f^{-1}(3)$ and $f\left(f^{-1}(2)\right)$.

Find $\left(f^{-1}\right)'(a)$ for $f(x) = x^3 + 3\sin x + 2\cos x$ and $a = 2$.

In-Class Discussion

6.1d

Exercise:

Let $f(x) = \sqrt{x-2}$ and $a = 2$.

a) Show that f is one-to-one

Find $\left(f^{-1}\right)'(a) = \dfrac{1}{f'\left(f^{-1}(a)\right)}$

In-Class Practice

1. Let $f(x) = \dfrac{1}{x-1}, \qquad x > 1, \qquad a = 2.$

a) Show that f is one-to-one.

b) Use Theorem 7 to find $(f^{-1})'(a)$.

c) Calculate $f^{-1}(x)$ and state the domain and range of f^{-1}.

d) Calculate $(f^{-1})'(a)$ from the formula in part (c) and check that it agrees with the result of part (b).

e) Sketch the graphs of f and f^{-1} on the same axes.

2. Find $(f^{-1})'(a)$ for $f(x) = \sqrt{x^3 + 4x + 4}, \qquad a = 3.$

3. If g is an increasing function such that $g(2) = 8$ and $g'(2) = 5$, calculate $(g^{-1})'(8)$.

4. If f is a one-to-one, twice differentiable function with inverse function g, show that

$$g''(x) = -\frac{f''(g(x))}{[f'(g(x))]^3}$$

What conclusion can you draw about the graph of $f^{-1}(x)$ if f is increasing and concave upward?