11.9 - Representations of Functions as Power Series

Preparation (Watch at Home)

11.9a

Exercises:

Find a power series representation for $f(x) = \dfrac{1}{1 - x}$.

$$\frac{1}{1 - x} = \sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + \cdots$$

Express $\dfrac{1}{1 + x^2}$ as a power series.

11.9b

Exercises:

Find a power series representation for the function and determine the interval of convergence.

$$f(x) = \frac{5}{1 - 4x^2}$$

$$f(x) = \frac{4}{2x + 3}$$

$$f(x) = \frac{x}{2x^2 + 1}$$

11.9c

More power series representations using term-by-term integration

$$\ln(1 + x) = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n} x^n$$

$$\tan^{-1} x = \sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1} x^{2n+1}$$

In-Class Discussion

11.9d

Exercises:

Find a power series representation for the function and determine the interval of convergence.

$f(x) = x^2 \tan^{-1}\left(x^3\right)$

$f(x) = \left(\dfrac{x}{2 - x}\right)^3$

11.9e

Exercise:

Use a power series to approximate the definite integral to six decimal places.

$\int_0^{0.2} x \ln\left(1 + x^2\right)\ dx$

In-Class Practice

1. Express the function as the sum of a power series by first using partial fractions. Find the interval of convergence.

$$f(x) = \frac{2x+3}{x^2+3x+2}$$

2. Find a power series representation for the function and determine the radius of convergence.

$$f(x) = \frac{x^2+x}{(1-x)^3}$$

3. Evaluate the indefinite integral as a power series. What is the radius of convergence?

$$\int \frac{\tan^{-1} x}{x}\, dx$$

4. Use a power series to approximate the definite integral to six decimal places.

$$\int_0^{1/2} \arctan \tfrac{x}{2}\, dx$$