11.8 - Power Series

Preparation (Watch at Home)

11.8a

Form of a power series

$$\sum_{n=0}^{\infty} a_n x^n = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \cdots$$

Exercise:

Find the radius and interval of convergence of $\sum_{n=0}^{\infty} (x+2)^n$.

Theorem (statement):

For a given power series $\sum_{n=0}^{\infty} a_n(x-a)^n$, there are only three possibilities:

i. The series converges only for $x = a$
$R = 0$; interval: $\{a\}$

ii. The series converges for all $x \in \mathbb{R}$
$R = \infty$; interval: $(-\infty, \infty)$

iii. There is a number $R > 0$ such that the series converges if $|x-a| < R$ and diverges if $|x-a| > R$

Note: Convergence at the endpoints of the interval is determined by testing them individually.

Exercises:

Find the radius of convergence and interval of convergence of the series.

11.8b

$$\sum_{n=1}^{\infty} \frac{(-1)^n x^n}{\sqrt[3]{n}}$$

$$\sum_{n=1}^{\infty} n^n x^n$$

$$\sum_{n=1}^{\infty} \frac{x^{2n}}{n!}$$

11.8c

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n\,5^n}\,x^n$$

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{(2n-1)2^n} (x-1)^n$$

In-Class Discussion

11.8d

$$\sum_{n=2}^{\infty} \frac{b^n}{\ln n} (x-a)^n,\ b > 0$$

11.8e

$$\sum_{n=1}^{\infty} \frac{n! x^n}{1 \cdot 3 \cdot 5 \cdot \cdots \cdot (2n-1)}$$

In-Class Practice

Find the radius of convergence and interval of convergence of the series.

1. $\sum_{n=1}^{\infty} \frac{(-1)^n x^n}{n^2}$

2. $\sum_{n=1}^{\infty} 2^n n^2 x^n$

3. $\sum_{n=1}^{\infty} \frac{\sqrt{n}}{8^n}(x+6)^n$

4. $\sum_{n=1}^{\infty} \frac{(2x-1)^n}{5^n \sqrt{n}}$

5. $\sum_{n=1}^{\infty} \frac{n^2 x^n}{2 \cdot 4 \cdot 6 \cdot \cdots \cdot (2n)}$