

11.8 - Power Series

Preparation (Watch at Home)

11.8a

Form of a power series

$$\sum_{n=0}^{\infty} a_n x^n = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \cdots$$

Exercise:

Find the radius and interval of convergence of $\sum_{n=0}^{\infty} (x + 2)^n$.

Theorem (statement):

For a given power series $\sum_{n=0}^{\infty} a_n(x - a)^n$, there are only three possibilities:

i. The series converges only for $x = a$

$R = 0$; interval: $\{a\}$

ii. The series converges for all $x \in \mathbb{R}$

$R = \infty$; interval: $(-\infty, \infty)$

iii. There is a number $R > 0$ such that the series converges if $|x - a| < R$ and diverges if $|x - a| > R$

Note: Convergence at the endpoints of the interval is determined by testing them individually.

Exercises:

Find the radius of convergence and interval of convergence of the series.

11.8b

$$\sum_{n=1}^{\infty} \frac{(-1)^n x^n}{\sqrt[3]{n}}$$

$$\sum_{n=1}^{\infty} n^n x^n$$

$$\sum_{n=1}^{\infty} \frac{x^{2n}}{n!}$$

11.8c

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n 5^n} x^n$$

This image shows a blank sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

In-Class Discussion

11.8d

$$\sum_{n=2}^{\infty} \frac{b^n}{\ln n} (x-a)^n, b > 0$$

[illegible]

11.8e

$$\sum_{n=1}^{\infty} \frac{n!x^n}{1 \cdot 3 \cdot 5 \cdot \dots \cdot (2n-1)}$$

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In-Class Practice

Find the radius of convergence and interval of convergence of the series.

This image shows a blank sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

$$3. \sum_{n=1}^{\infty} \frac{\sqrt{n}}{8^n} (x+6)^n$$

$$4. \sum_{n=1}^{\infty} \frac{(2x-1)^n}{5^n \sqrt{n}}$$

$$5. \sum_{n=1}^{\infty} \frac{n^2 x^n}{2 \cdot 4 \cdot 6 \cdot \dots \cdot (2n)}$$
