

11.5 - Alternating Series

Preparation (Watch at Home)

11.5a

The Alternating Series Test (statement and proof):

If the alternating series

$$\sum_{n=1}^{\infty} (-1)^{n-1} b_n = b_1 - b_2 + b_3 - b_4 + \cdots, \text{ where } b_n > 0, \text{ satisfies}$$

i. $b_{n+1} \leq b_n$ for all n

ii. $\lim_{n \rightarrow \infty} b_n = 0$

then the series is convergent.

[illegible]

[illegible]

11.5b

Exercises:

Test the series for convergence or divergence.

$$\sum_{n=0}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n+1}}$$

$$\sum_{n=1}^{\infty} (-1)^{n+1} n e^{-n}$$

$$\sum_{n=1}^{\infty} (-1)^{n-1} \arctan n$$

11.5c

Theorem (statement and proof):

The Alternating Series Estimation Theorem: If $s = \sum_{n=1}^{\infty} (-1)^{n-1} b_n$ is a series that converges by the Alternating Series Test, then $|R_n| \leq b_{n+1}$.

[illegible]

Exercise:

Use the Alternating Series Estimation Theorem to estimate the sum correct to four decimal places.

$$\sum_{n=1}^{\infty} (-1)^{n-1} \frac{n}{8^n}$$

11.5d

Definitions: (absolute and conditional convergence)

A series $\sum a_n$ is called absolutely convergent if the series of absolute values

$\sum |a_n|$ is convergent.

A series $\sum a_n$ is conditionally convergent if it is convergent but the series of absolute values $\sum |a_n|$ is divergent.

Theorem (statement and proof):

If a series $\sum a_n$ is absolutely convergent, then it is convergent.

In-Class Practice

Test the series for convergence or divergence.

1.
$$\sum_{n=1}^{\infty} (-1)^n \frac{n^2}{n^2 + n + 1}$$

$$2. \sum_{n=1}^{\infty} (-1)^n \frac{\sqrt{n}}{2n+3}$$

$$3. \sum_{n=1}^{\infty} \frac{n \cos n\pi}{2^n}$$

$$4. \sum_{n=1}^{\infty} (-1)^n (\sqrt{n+1} - \sqrt{n})$$

5. Show that the series $\sum_{n=1}^{\infty} \frac{\left(-\frac{1}{3}\right)^n}{n}$ is convergent. How many terms of the series do we need to add in order to find the sum so that $|\text{error}| < 0.0005$?
