11.4 - The Comparison Tests

Preparation (Watch at Home)

11.4a

The Comparison Test:

Suppose that $\sum a_n$ and $\sum b_n$ are series with positive terms.

i. If $\sum b_n$ is convergent and $a_n \leq b_n$ for all n, then $\sum a_n$ is also convergent.

ii. If $\sum b_n$ is divergent and $b_n \leq a_n$ for all n, then $\sum a_n$ is also divergent.

Exercise:

Determine whether the series converges or diverges.

$$\sum_{n=2}^{\infty} \frac{1}{\sqrt{n}-1}$$

11.4b

The Limit Comparison Test:

Suppose that $\sum a_n$ and $\sum b_n$ are series with positive terms. If $\lim_{n\to\infty} \frac{a_n}{b_n} = c$, where c is a positive, finite number, then either both series converge or both series diverge.

Two special cases:

If $\lim\limits_{n\to\infty} \dfrac{a_n}{b_n} = 0$ AND if $\sum b_n$ converges, then $\sum a_n$ also converges.

If $\lim\limits_{n\to\infty} \dfrac{a_n}{b_n} = \infty$ AND if $\sum b_n$ diverges, then $\sum a_n$ also diverges.

Exercises:

Determine whether the series converges or diverges.

11.4d

$$\sum_{n=1}^{\infty} \frac{6^n}{5^n - 1}$$

$$\sum_{k=1}^{\infty} \frac{k \sin^2 k}{1 + k^3}$$

$$\sum_{n=1}^{\infty} \frac{2}{\sqrt{n}+2}$$

In-Class Discussion

11.4c

$$\sum_{n=3}^{\infty} \frac{n+2}{(n+1)^3}$$

In-Class Practice

Determine whether the series converges or diverges.

1. $\displaystyle\sum_{n=1}^{\infty} \frac{n-1}{n^3-1}$

2. $\displaystyle\sum_{n=1}^{\infty} \frac{(2k-1)\left(k^2-1\right)}{(k+1)\left(k^2+4\right)^2}$

3. $\displaystyle\sum_{n=1}^{\infty} \frac{n^2+n+1}{n^4+n^2}$

4. $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n\sqrt{n^2-1}}$