11.2 - Series

Preparation (Watch at Home)

11.2a

Introduction to series

Partial sums

$s_n = a_1 + a_2 + a_3 + \cdots + a_n$

Sequence of partial sums

$\{s_n\} = s_1,\ s_2,\ s_3, \cdots, s_n$

Convergence of a series

Definition: If the sequence $\{s_n\}$ is convergent and the limit $\lim_{n\to\infty} s_n = s$ exists as a finite real number then the associated series $\sum a_n$ is **convergent**, and the sum of the series is $\sum_{n=1}^{\infty} a_n = s$.

Convergence of a geometric series

$\sum_{n=1}^{\infty} a\, r^{n-1}$ is convergent if $|r| < 1$ and divergent otherwise. If $|r| < 1$, then $S = \frac{a}{1-r}$.

11.2b

Theorem (statement and proof):

If $\sum_{n=1}^{\infty} a_n$ is convergent, then $\lim_{n\to\infty} a_n = 0$.

Divergence Test:

If $\lim_{n\to\infty} a_n$ does not exist or if $\lim_{n\to\infty} a_n \neq 0$, then the series $\sum_{n=1}^{\infty} a_n$ is divergent.

11.2c

Exercises:

Determine whether the series is convergent or divergent. If it is convergent, find its sum.

$$\frac{1}{3}+\frac{2}{9}+\frac{1}{27}+\frac{1}{243}+\frac{2}{729}+\cdots$$

Determine whether the series is convergent or divergent by expressing s_n as a telescoping sum. If it is convergent, find its sum.

$$\sum_{n=1}^{\infty} \ln \frac{n}{n+1}$$

In-Class Discussion

11.2d

Exercises:

Find the values of x for which the series converges. Find the sum of the series for those values of x.

$\sum_{n=1}^{\infty}(x+2)^n$

Express the number as a ratio of integers.

$10.1\overline{35} = 10.135353535\ldots$

A patient is injected with a drug every 12 hours. Immediately before each injection the concentration of the drug has been reduced by 90% and the new dose increases the concentration by 1.5 mg/L.

What is the concentration after three doses?

If C_n is the concentration after the nth dose, find a formula for C_n as a function of n.

What is the limiting value of the concentration?

In-Class Practice

1. Determine whether the geometric series is convergent or divergent. If it is convergent, find its sum.

$$\sum_{n=1}^{\infty} \frac{6 \cdot 2^{2n-1}}{3^n}$$

Determine whether the series is convergent or divergent. If it is convergent, find its sum.

2. $\sum_{n=0}^{\infty} [(-0.2)^n + (0.6)^{n-1}]$

3. $\sum_{n=1}^{\infty} \frac{2^n + 4^n}{e^n}$

4. Find the sum of the series.

$\sum_{n=2}^{\infty} \frac{1}{n(n+3)}$

5. Find the values of x for which the series converges. Find the sum of the series for those values of x.

$$\sum_{n=0}^{\infty} (-4)^n (x-5)^n$$